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THE EFFECT OF THE END CONNECTIONS ON THE DISTRIBUTION OF STRESS IN CERTAIN TENSION MEMBERS.[1]

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INTRODUCTION.

THE experimental study of the distribution of stress in structural members has received considerable attention in recent years. The experiments of American investigators upon actual structures are well known. Most of these investigations, however, have been made with strain gauges, such as the Howard and Berry, which, while excellently adapted to the more or less rough determination of average strains in members of actual structures, are scarcely suited to refined measurement of the distribution of strain in a member subjected to stresses which vary considerably over the cross-section. Thus the analysis of the distribution of stress in single or built-up structural members and its modification due to different types of end connections has scarcely received the attention which it deserves from its practical importance. The optical determination of complex stress distribution in transparent materials by the use of polarized light as developed by Coker, Mesnager, Hönigsberg and others,[2] although giving accurate results within its range, does not appear to be

[1] Communicated by Professor H. M. Mackay.

[2] See British Association Report, 1914.

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capable of extension to the investigation of built-up members, as the actual end conditions, connections between the parts, etc., cannot be correctly imitated in glass or xylonite models. The thermo-electric method would appear to be more promising, but has not yet been sufficiently investigated. For the present it would seem as if reliance must be placed upon measurement with some form of extensometer. Most extensometers, however, measure only the average stress over a considerable length, are bulky and inconvenient, and cannot be used in positions difficult of access, such as the inner portions of a built-up member.

Some time ago the writer read a paper before the Canadian Society of Civil Engineers [3] in which he described investigations of the distribution of stress in certain members made with a simplified form of Martens' mirror extensometer constructed in the laboratories of McGill University, Montreal. This instrument, extremely simple in construction and operation, was shown to be capable, when certain precautions are observed in its use, of measuring strains accurately to $\frac{1}{100.000}$ inch on a length as small as 2 inches, and of being used in the most confined positions, such, for example, as the space between two angles placed back to back and separated by as little as $\frac{3}{8}$ inch. It was thus proved to be eminently suitable for the laboratory investigation of strain distribution in built-up members.

In the paper cited above, the distribution of stress in single- and double-angle tension members riveted to end plates was considered. It was shown that the assumption of planar distribution of stress over the cross-section was justified and that the actual distribution in a single-angle member was closely in accordance with that given by the theory of eccentric stresses developed by Bach, Müller-Breslau and others, and put into workable form by Professor L. J. Johnson, of Harvard.[4] It was also shown that a double-angle member did not act as one piece, but that each angle bent about its own neutral axis, and thus the correct way of considering a built-up member was not to regard it as a single piece bending as a beam, but as several pieces, each trying to bend about its own neutral axis, but restrained by the subsidiary members, such as tie plates or latticing.

In these experiments 3 inch × 3 inch × ¼ inch angles were

[3] *Transactions Canadian Society of Civil Engineers*, vol. xxvi, p. 224.

[4] See *Trans. Am. Soc. Civ. Eng.*, 1906, p. 169.

used, riveted to end plates of the same width, that being the greatest which would fit into the grips of the testing machine. In this case the effect of the end constraints was found to be very little in the case of the single angle, but it was thought advisable to repeat the experiments with end plates more like the gusset plates to which such members are usually attached in practice. To this end special grips were made so that plates of considerable size could be used, and the results of the experiments to be described later show that there must be considerable end restraint under practical conditions. In the present paper the effect of this constraint is considered, and also the effect of changing the axis of pull on the gusset plate, but the chief object is the investigation of the effect of lock angles at the ends of the members. In practice the ends of single- and double-angle members are usually secured to the gusset plates directly by a row of rivets between the plate and the angle and indirectly through the medium of a small angle riveted both to the gusset plate and to the member as shown in Fig. 5, which represents the experimental specimens. The function of this small angle, called a "lock" or "lug" angle, is supposed to be two-fold. First, by transmitting a pull through both legs of the angle or angles forming the member, it is supposed to lessen the eccentricity of connection, so that often the stress is calculated as though uniformly distributed over the cross-section; secondly, by allowing more rivets to be used at the ends of the member, it is supposed to be equivalent to lengthening the end connections. The writer hopes to show that both of these claims are to a large extent unfounded; that lock angles really serve very little useful purpose, and that the slight effect which they have, helpful or otherwise, cannot be predetermined.

Before entering upon a discussion of the experiments a brief account of the theory upon which the analysis of the results is based will be given.

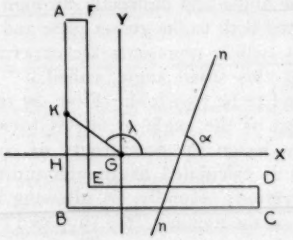
PART I.—THEORETICAL.

§1. *The Distribution of Stress in Eccentrically Loaded Members of Uniform Cross-section.*

A single angle such as is shown in Fig. 5, loaded in tension or compression through rivets in one leg, is an example of an eccentrically loaded member in which the load axis does not lie

in an axis of symmetry of the cross-section. If the ends of the member are not appreciably restrained from bending by the end connections, the axis of loading may be said to lie along the line of rivets and at the common face of the end plate and the angle. This axis is represented by K in Fig. 1, which represents a cross-section of the member. It will readily be seen that the ordinary theory of eccentric loading, which is true only for loads applied in an axis of symmetry of the cross-section, cannot be applied to this case, and recourse must be had to the general theory already mentioned.[5] For a full account of this theory the reader is referred to the paper by Professor L. J. Johnson referred to above or to Appendix I of the paper by the present writer in the *Transactions of the Canadian Society of Civil Engineers*, vol. xxvi, p. 224. A brief *résumé* of the

FIG. 1.



theory will be given here, as it is essential to the understanding of the experimental results. Referring to Fig. 1, let G represent the centroid of the cross-section and Gx and Gy be a pair of co-ordinate axes parallel to the legs of the angle. Then, if N be the normal force applied at K, the stress at any point of the cross-section will be equal to $\frac{N}{A}$, where A is the area of the section, together with a stress arising from the bending moment $N.KG$ to which the angle is subjected. This moment will cause bending about a neutral axis nn, which will not, as in the case of loading in a plane of symmetry, be perpendicular to KG, but at an angle a to Gx, given by the equation

$$\tan a = \frac{I_x - J \tan \lambda}{J - I_y \tan \lambda} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

[5] L. J. Johnson, *loc. cit.*

where I_x and I_y are the moments of inertia of the section about Gx and Gy respectively and J is the product of inertia of the section with respect to the axes Gx and Gy,—*i.e.*, the $\int\int xydxdy$ with respect to these axes.[6] The above equation is deduced by equating the sum of the moments of the stresses over the section about Gx and Gy respectively to the components of the bending moment $N.KG$ about these axes, assuming that the distribution of stress over the section follows a linear law.[7] The stress at any point (xy) of the section may be shown to be given by either of the equations

$$f = N\left[\frac{1}{A} + \frac{y - x\tan\alpha}{J - I_y\tan\alpha}\cdot x_k\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$= N\left[\frac{1}{A} + \frac{y - x\tan\alpha}{I_x - J\tan\alpha}\cdot y_k\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

where (x_k, y_k) are the coördinates of the load axis K. In order to find the maximum stress, it is only necessary to substitute for (xy) the coördinates of the point most distant from the neutral axis. If f be equated to zero in either of the equations (2) or (3), the equation of the neutral axis nn will be obtained.

§2. *The S-Polygon.*

The S-polygon, a modification of the W-Fläche of Land, was first introduced by L. J. Johnson.[8] It is a figure which gives at a glance the point of maximum bending stress and the value of the latter for any given load axis, and, as its use will facilitate certain deductions to be made later, its construction will be considered briefly here.

For the bending of ordinary I sections the steel handbooks give a quantity called the " modulus " of the section which is equal to $\frac{M}{f} = \frac{I}{y}$ where M is the bending moment applied to the section, y the distance of the skin from the neutral axis, f the maximum stress and I the moment of inertia of the section about the neutral axis. It may be defined as that quantity by which the bending moment at any section must be divided in order to give the maximum

[6] I_x and I_y are given in the steel handbooks for all ordinary sections, but J has to be calculated. For the method of calculation see either of the papers cited above.

[7] This was shown to be the case experimentally by the writer, *loc. cit.*

[8] *Loc. cit.*

stress at the section. In the general case of flexure as considered above there will be a similar quantity which may be termed the "flexure modulus" (S) of the section. Thus $S=\frac{M}{f}$. The bending moment on the section is $N.KG$. The stress due to this at any point of the section is, from equation (2), given by

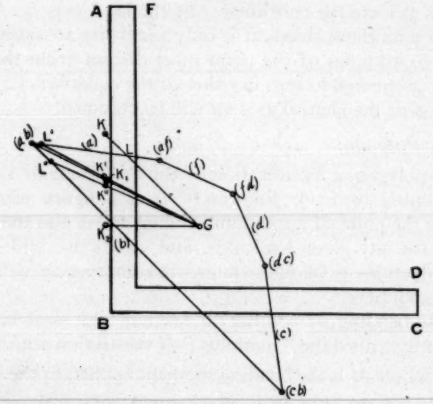
$$f_b = \frac{y - x\tan\alpha}{J - I_y\tan\alpha} \cdot NG\cos\lambda \,. \quad \ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Eliminating α between this equation and equation (1), and rearranging terms, the section modulus for (xy) is given by

$$S = \frac{M}{f} = \frac{I_x I_y - J^2}{(yI_y - xJ)\sin\lambda + (xI_x - yJ)\cos\lambda} \cdot \ldots\ldots\ldots (5)$$

If the point (xy) remains fixed while λ changes, the above is readily seen to be the polar equation of a straight line, having

FIG. 2.



radius vector S and angle λ. This line may be termed the S-line for the point (xy). If S-lines be drawn for all points at which the maximum stress may occur, a polygon, called the S-polygon, will be obtained. Thus, for example, in the case of the single angle Fig. 2, the maximum stress may occur at A, B, C, D, or F. If G be chosen as the pole and Gx as the initial line, the figure (ab), (bc), (cd), etc., is the S-polygon drawn with S to the same scale as the linear dimensions of the figure. Thus for the load

point K, GL represents S and the maximum stress occurs at B. But $S = \frac{M}{f_b}$ and $M = N.KG$, therefore f_b, the bending stress at B, is equal to

$$\frac{M}{S} = N \cdot \frac{KG}{LG} .$$

Thus the total stress at B, which is the maximum stress at the section, is given by

$$f = \frac{N}{A} + N \cdot \frac{KG}{LG} . \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

In order that this may be true, it is, of course, necessary that the scale of the S-polygon should be the same as the linear scale of the figure and that the origin be G and the initial line Gx.

§3. *The Construction of the S-Polygon.*

The best method of constructing the S-polygon of an angle is by locating the apices. Each apex corresponds to a side. It may be shown by transforming equation (5) into rectilinear coördinates, substituting the coördinates of any two points $(x_a\ y_a)$, $(x_b\ y_b)$, and solving for the apex (ab) of the S-polygon, that

$$\left.\begin{aligned} x_{ab} &= \frac{(x_a - x_b)\,J - (y_a - y_b)\,I_y}{x_a y_b - x_b y_a} , \\ y_{ab} &= \frac{(x_a - x_b)\,I_x - (y_a - y_b)\,J}{x_a y_b - x_b y_a} . \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (7)$$

If the side considered be parallel to Gx, so that $y_a = y_b$, the above equation gives

$$x_{ab} = \frac{J}{y_a} , \quad y_{ab} = \frac{I_x}{y_a} ,$$

while, if the side be parallel to Gy,

$$x_{ab} = \frac{I_y}{x_a} , \quad y_{ab} = \frac{J}{x_a} .$$

Considering, for example, the single-angle specimen I, used in the experiments to be described later and for which $I_x = I_y = 1.37$ (in.)^4, $J = -0.81$, and the distance from the centroid to the back of the angle is 0.85″, the apex of the S-polygon corresponding to the side AB has the coördinates

$$x_{ab} = -\frac{1.37}{0.85} = -1.61, \quad y_{ab} = \frac{0.81}{0.85} = 0.95.$$

The other apices may be determined in a similar manner, and the complete polygon is shown in Fig. 2.

The S-polygon furnishes the simplest method of determining the ratio of maximum to mean stress when a number of load axes have to be considered. It may also be used to establish a number of interesting results. For instance, as the point K moves down AB, starting from A, the radius vector GL terminates first on the f line, then on the a line, and finally on the b line, showing that the point of maximum stress changes from F to A and then to B. The ratio $\frac{GK}{GL}$ also changes considerably. It will be shown later that the S-polygon is of material aid in analyzing the probable effect of a lock angle or a change in the restraining couple.

§4. *The Effect of Lateral Deflection and of End Restraints.*

The earlier experiments of the writer have shown that in the case of a single angle loaded in tension and not effectively restrained at the ends, the axis K is along the line of rivets and slightly within the loading plate. In a long member there will be a measurable lateral deflection as the load is applied, the centroid of each section trying to set itself in the load axis. This will cause a change in the position of K relative to the section and consequently affect the distribution of stress over the section. This effect is not usually great enough to be of much practical importance, but must be considered in reducing experimental results. Of much greater interest is the effect of the gusset plate and lock angle in affecting the position of the load axis. A stiff end connection will introduce constraining couples both in and at right angles to the plane of the end plate, and these may have important effects upon the distribution of stress. A lock angle is often supposed to so constrain the ends that the eccentricity of pull may be neglected, although how far this is from actually being the case will be shown later. The chief object of the present paper is the investigation of these different constraints in the case of single- and double-angle members.

§5. *The Working Loads upon Eccentrically Loaded Members.*

In a compression member it is, of course, always unsafe to allow the stress at any point to exceed the elastic limit of the material, even though the load which would cause this stress be well within the theoretical buckling load for a long column. Experiment has shown, however, that, in the case of a tension member, a redistribution of stress usually occurs under such con-

ditions, and that the elastic limit may be considerably exceeded at certain parts of the section without danger to the structure as a whole. This would lead to the supposition that, when the stresses arising from eccentricity are taken into account, higher working stresses than usual might be used. It would, however, be unsafe to proceed on this assumption without more experimental evidence as to whether, in such case, the new distribution of stress persists without alteration after many reloadings. At present it would seem to be unwise to allow the stress over any considerable part of the structure to rise above the ordinary working stress of the material at any point of the cross-section, especially considering that there are always higher stresses locally near to the end connections. Thus it would appear that single-angle members, and to some extent double-angle members, should always be designed for the maximum stress, allowing for eccentricity,—*i.e.*, including what are often termed "secondary" stresses, although they are frequently quite as large as, or even larger than, the so-called "primary" or uniform stress, leaving what may be termed the "tertiary" or local stresses due to the end connections to be included in the factor of safety.

PART II.—THE EXPERIMENTS.

§1. *The Extensometers.*[9]

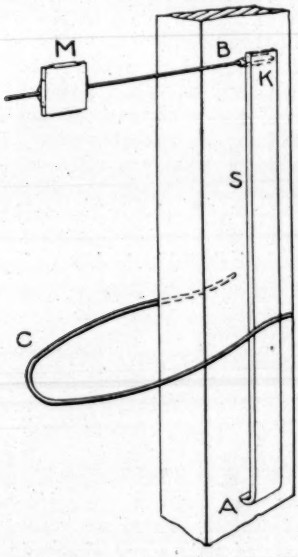
The extensometers used were a simplified form of the Martens' type, designed and constructed in the McGill Testing Laboratory, where they have been in use since 1906, and have been proved capable of giving very accurate results.

The principle of the instrument is shown in Fig. 3. It consists essentially of a double knife-edge, K, which fits between the specimen under test and a V groove in one end of a steel strip S, which is in contact with the specimen at A, and is pressed against it by means of a clip C. A change in the length of the specimen between A and B causes the knife-edge to tilt, and the tilt is measured by means of a telescope and scale, the scale being reflected in a mirror M attached to the knife-edge. In the actual instrument the steel strip is $\frac{3}{8}$ inch wide, $\frac{1}{8}$ inch thick, the length $A\ B$ being chosen to suit requirements. The end A is is turned at right angles and brought to a sharp edge so that it may not slip on the specimen. The knife-edge is of hardened steel

[9] This section is reprinted from the writer's earlier paper, *loc. cit.*

about 0.18 inch × 0.12 inch × 0.45 inch, and the mirror is attached by means of a piece of steel knitting needle. The mirror is held in a clip of thin sheet steel which is arranged so that it can slide and rotate on the needle, a thin copper strip protecting its back from injury. This clip permits of a small amount of lateral adjustment. The mirror is about ½ inch square and must be as truly plane as possible, as otherwise there will be an error

FIG. 3.

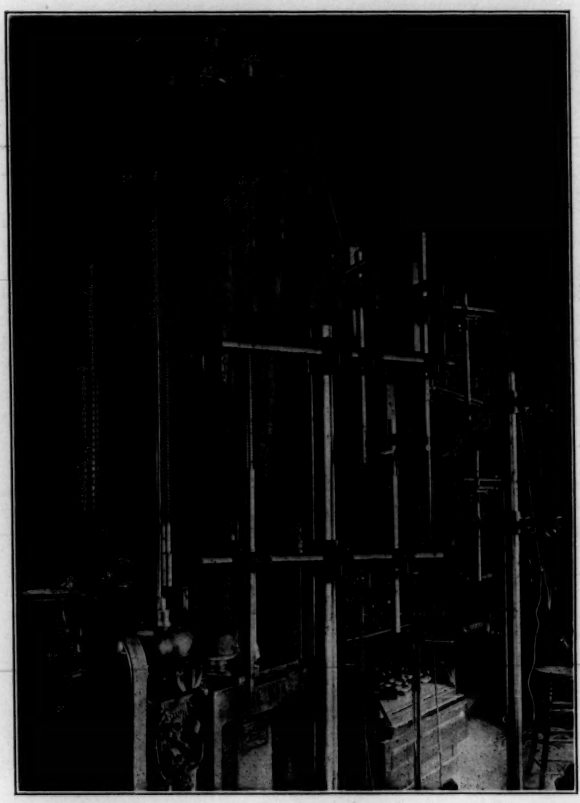


introduced when the image of the scale moves to a different part of its surface, as it must do if the specimen deflects at all during test. In the original form of Martens' extensometer there was a device for adjusting the mirror and also a balance weight at the opposite side of the knife-edge, but these refinements are not only unnecessary but cumbersome, and make the instrument less adapted to use in restricted positions.

The extensometer is calibrated in a Whitworth measuring

machine, and a calibrating rod is prepared for each instrument, giving the distance from the scale to the mirror, so that a definite

FIG. 4.



distance on the scale may correspond to a given extension or compression on the specimen. In the case of the experiments described below, ½ inch on the scale, subdivided into ten equal

divisions, corresponded to $\frac{1}{1000}$ inch, so that the change of length of the specimens was easily read to $\frac{1}{100.000}$ inch. The length of the rod was about 4 feet, varying with different instruments. The angle turned through by the mirror in any test is so small that there is no appreciable error in using a straight scale for the readings. This was verified by turning the mirror in the Whitworth measuring machine through much greater angles than those through which it turns in the tests. It was also found that different strips (S) did not affect the calibration, so that a knife-edge could be used with different lengths of strip without recalibration. It was estimated that, under the conditions of test, the instrument reads accurately to $\frac{1}{100.000}$ inch.

The kind of telescope used affects greatly the facility with which readings may be taken. The McGill Testing Laboratory telescopes are adjustable vertically and horizontally, besides moving bodily about a vertical axis. They are carried on specially-made stands permitting any kind of adjustment to be made with ease (see Fig. 4). The extensometer must be carefully used in order to give correct results. The mirror should be, in its mean position, parallel to the scale and the telescope should be opposite to the mirror. The clip must be arranged so that the knife-edge is held quite firmly, otherwise it will not tilt correctly. The best clips are made from pieces of copper wire.

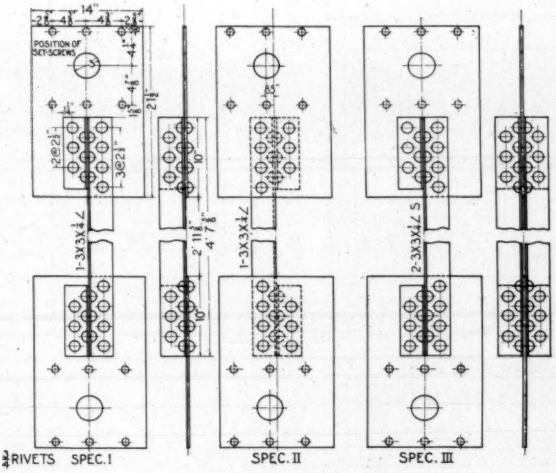
If the direction of $A\ B$ remains unchanged during test, the difference of the scale reading between two loads will be an accurate measure of the strain of $A\ B$ for the given load difference, but if $A\ B$ alters in direction this will not be the case. If, however, two readings are taken, one with the extensometer in the position shown, and the other with the knife-edge at A and the sharp edge of the strip at B, the mean of the two will be correct. When any doubt exists it is always better to do this so as to eliminate possible error.

In the opinion of all who have worked with these instruments at McGill University they are the most simple, practicable, and accurate extensometers in use. It will be seen that they may be readily used in the most restricted positions, as, for instance, between the two angles of the double-angle members, where the width is only ⅜ inch. Fig. 4 shows the general arrangement of the apparatus.

§2. *The Specimens and the Method of Holding Them.*

Experiments were made on the three specimens, two single-angle members and one double-angle, shown in Fig. 5. All were ordinary shop products made by the Dominion Bridge Company of Montreal, and the angles were of uniform section 3 inches × 3 inches × ¼ inch. Careful measurements showed that the section varied a little in the three specimens, and the actual areas, moments of inertia, etc., were computed and are given at the

FIG. 5.



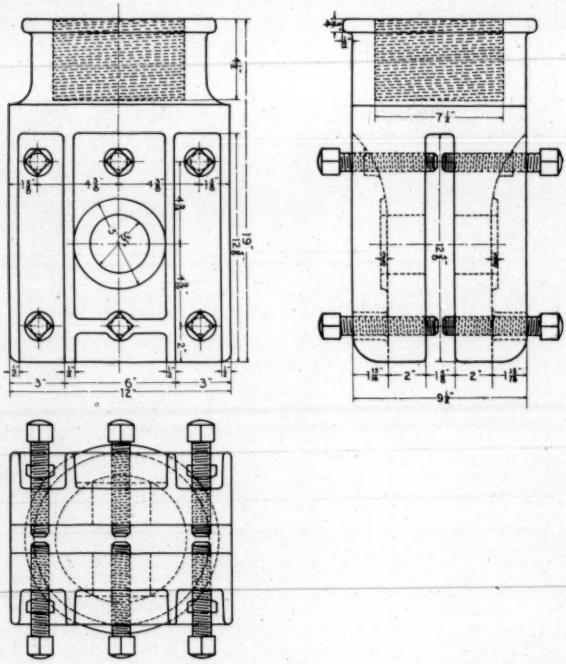
Specimens used in the experiments.

heads of Tables I, II, and III, pp. 153–155. The members were of a uniform length of 55⅝ inches over all and 35⅝ inches between the end plates, and were secured to the latter by means of four ¾-inch rivets having a pitch of 2½ inches and by lock angles riveted to the plate by three ¾-inch rivets and to the outstanding leg of the angle by a like number.

In the earlier experiments referred to above, similar angles were used, without lock angles, riveted to end plates of the same width, 3 inches, as the angles, secured directly in the grips of the

testing machine, the distance from the end of the grips to the end of the angle being about 5½ inches. With these end plates it was found that the restraining couple upon the angle was very small, and it was thought desirable, in the present tests, to substitute end connections more nearly similar to those met with in practice,

FIG. 6.

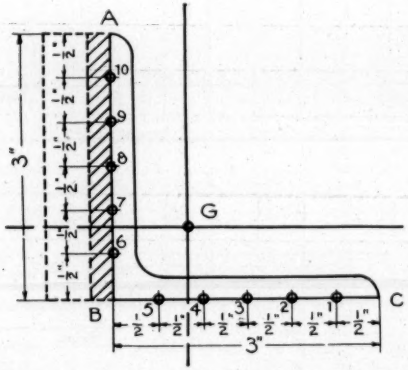


i.e., wide plates firmly secured. To this end special grips were designed to fit on the jaws of the machine. These are shown in Fig. 6. They were steel castings. The end plates, which were 1 foot 2 inches wide and ⅜ inch thick, were held by steel pins 3 inches in diameter fitting tightly into bushes in the castings, and

were restrained from turning by six set screws on each side, as indicated in Fig. 6. The distance from the end of the angle to the set screws was approximately 2 inches, so that the ends of the members were quite as effectively held as in most practical cases.

The position of the pin with respect to the rivets was made different in the different specimens. In Specimens I and III it was in line with the back of the main angle, while in Specimen II it was in line with the centroid of the main angle. The object of

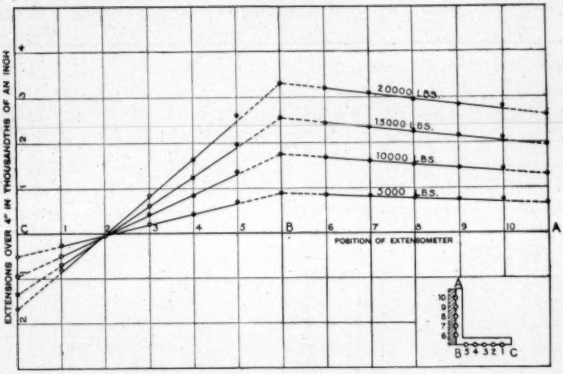
FIG. 7.



this was to find the effect of a change in the line of pull on the gusset plate,— *i.e.*, of different eccentricity of end connection. Further changes were also made during the tests, as will be described later. The specimens were also tested with the lock angles removed in order to study the action of the latter.

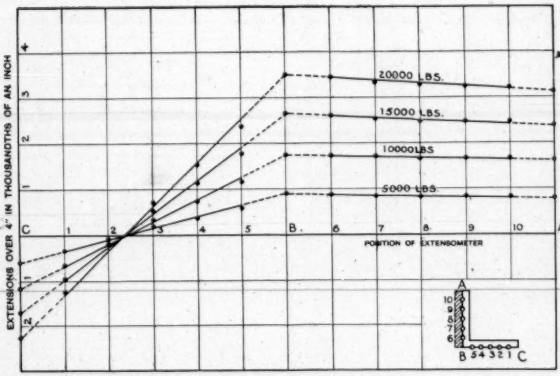
The machine used was the Emery testing machine in the Testing Laboratory of McGill University. This machine is of the vertical type and has a capacity of 150,000 pounds. It is eminently suited to this kind of work as the line of pull, suitable means being taken to steady the straining head, remains constant, and there is an entire absence of vibration.

FIG. 8.



Specimen I with lock angle. Central section.

FIG. 9.

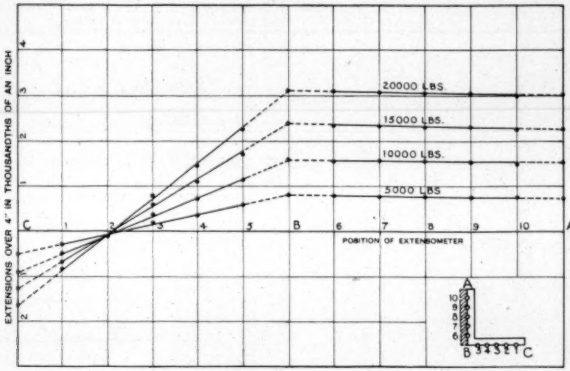


Specimen I with lock angle. End section.

§3. *The Tests.*

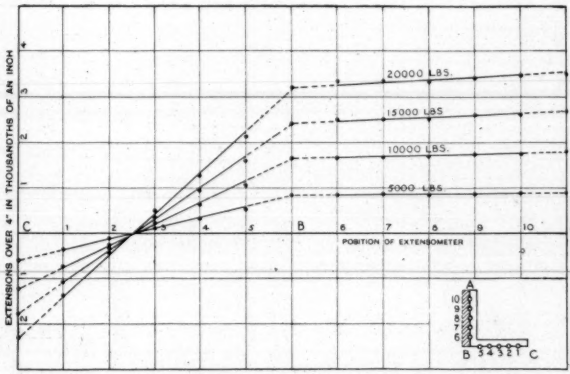
The experiments were directed to a determination of the distribution of stress under different loads at the central cross-sections of the specimens and also at sections near to the end plates.

FIG. 10.



Specimen I. Lock angle removed. Central section.

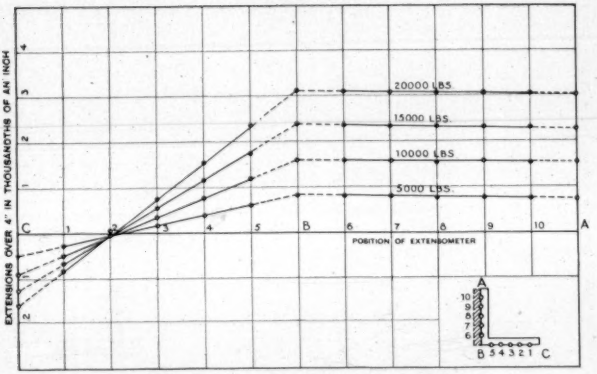
FIG. 11.



Specimen I. Lock angle removed. End section.

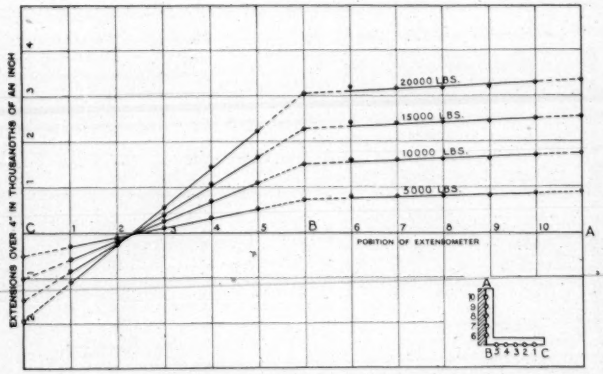
To this end extensometers were used at these sections with their distance pieces set parallel to the axis of pull. From the readings of these the longitudinal strains were found, and these were

FIG. 12.



Specimen II. With lock angle. End section.

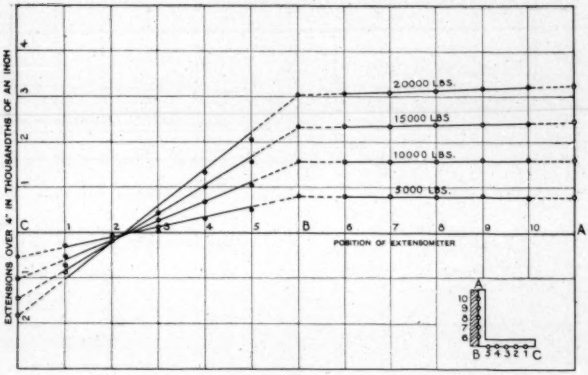
FIG. 13.



Specimen II with lock angle. End section.

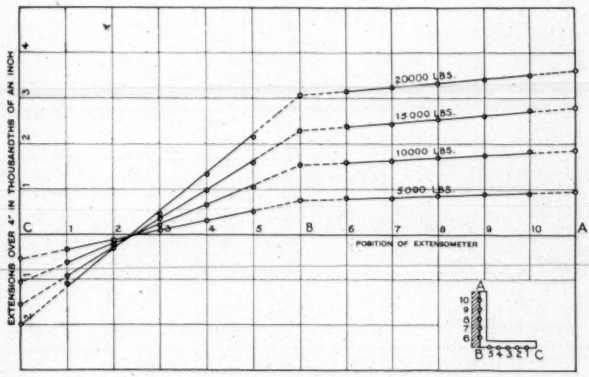
taken as being proportional to the longitudinal stresses, the effect of the strains perpendicular to the axis being so small as to be negligible. The distance pieces of the extensometers were 4

FIG. 14.



Specimen II. Lock angle removed. Central station.

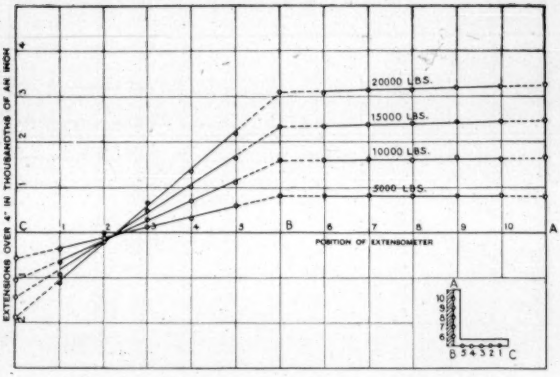
FIG. 15.



Specimen II. Lock angle removed. End section.

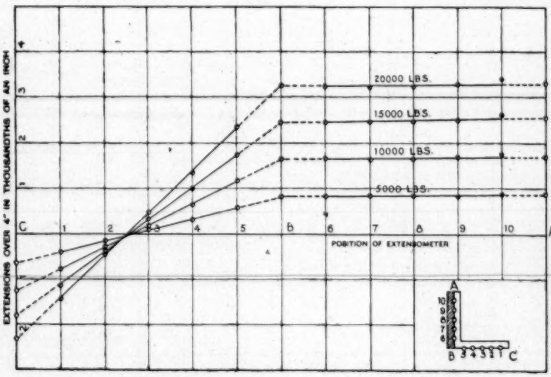
inches long, so that the strains measured were mean strains over that length. They could be considered as giving the distribution of stress over the central cross-section of the 4-inch range without

FIG. 16.



Specimen II. Line of pull in line of rivets. Central section.

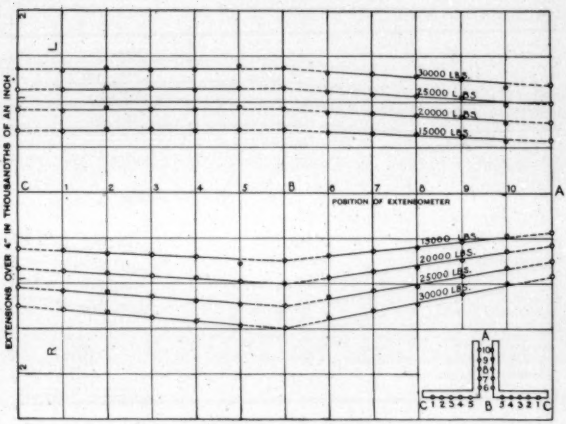
FIG. 17.



Specimen II. Line of pull in line of rivets. End section.

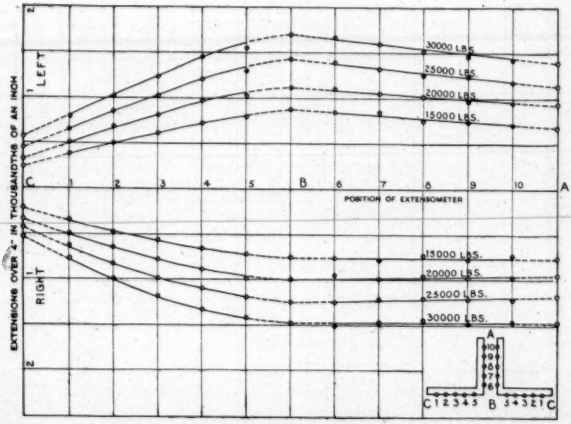
error at the central section, and to a fair degree of approximation at the ends. Readings of the extensometers were taken at half-inch intervals over the outside face of the angles, as shown in

FIG. 18.



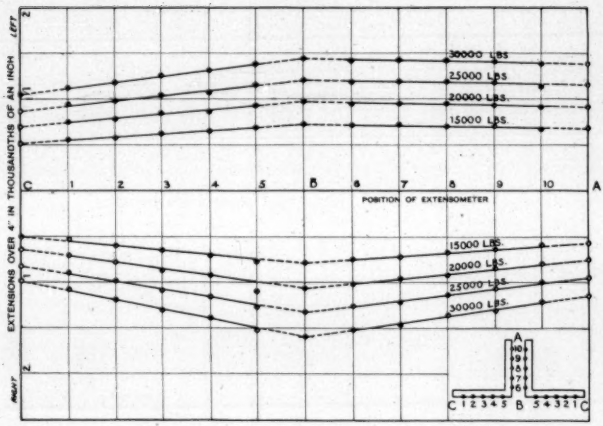
Double angle. Central station.

FIG. 19.



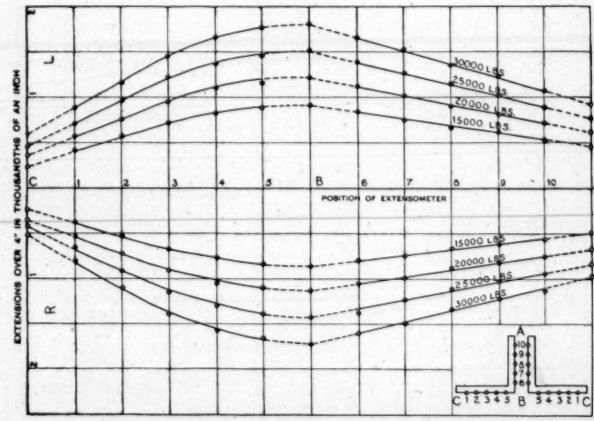
Double angle. End section.

FIG. 20.



Double angle without lock angles. Central section.

FIG. 21.



Double angle. Lock angles removed. End section.

Fig. 7, the extensometers being arranged first with the mirror at the lower end and then at the upper end and the mean of these readings taken, so that errors due to bending of the specimen were eliminated, as described above.

All the tests were carried out in a uniform manner. Two or more extensometers being set in position, an initial load of 100 pounds was applied to the specimen; the load was increased to its maximum value several times and then brought back to its initial value. The zeros of the extensometers were then set and readings taken at 5000, 10,000, 15,000, and 20,000 pounds respectively in the case of the single-angles and 15,000, 20,000, 25,000, and 30,000 pounds respectively in the case of the double-angle. The load was then brought back to its initial value and the zeros of the extensometers checked. No sets were allowed to pass in which the extensometers failed to return to their initial readings. All observations were repeated at least once before the extensometers were removed to other positions.

The uniformity of the results was remarkable. It was found that the specimen could be taken out of the machine and replaced without the extensometer readings for a given load being appreciably altered.

The lateral deflections of the specimens were carefully measured by means of telescopes and scales reading to $\frac{1}{100}$ inch. These deflections were allowed for in reducing the results, as will be explained later.

The Emery machine was calibrated by levers during the period of the tests and was found to be reading correctly to within 1 per cent. at all loads.

The sequence of tests was as follows:

(1) Specimen I.—Single, 3 inches × 3 inches × ¼ inch angle with lock angle. Line of pull on end plate in line with back of main angle.
(2) Same specimen, with lock angle removed.
(3) Specimen II.—Single, 3 inches × 3 inches × ¼ inch angle with lock angle. Line of pull on end plate in line with centroid of main section.
(4) Same specimen with lock angle removed.
(5) Same specimen with line of pull on end plate changed so as to be in line with rivets.

(6) Specimen III.—Double, 3 inches × 3 inches × ¼ inch angle section, space of ⅜ inch between angles, with lock angles. Line of pull in line with unconnected legs of main angles.

(7) Same specimen with lock angles removed.

The stresses corresponding to the extensometer readings are given in Tables I to III, pp. 153–155, the values of E being calculated from the readings and the total load, as will be described later. The curves of stress distribution are shown in Figs. 8 to 21.

PART III.—ANALYSIS OF THE EXPERIMENTAL RESULTS.

§1. *The Planar Distribution of Stress.*

The theory described in Part I of this paper is based upon the assumption that the distribution of stress over a cross-section normal to the axis of load follows a linear law. That this is true for members of the type considered in this paper has already been shown by the writer.[10] It is also evident from the curves shown in Figs. 8 to 21. These represent the actual mean extensometer readings for the various cases, the mean straight line through the experimental points being drawn and continued so as to give the maximum strains, which occur at one or other of the corners of the angles. It will be noticed that in every case the mean straight lines for readings 1 to 5, on being produced to the corner of the angle, agree exactly with those for readings 6 to 10. The deviations of the experimental points from the straight lines are in nearly all cases so small as to be unimportant, especially considering that the specimens were ordinary shop products. Where deviations do occur they are usually regular and apparently denote a slight actual departure from planar distribution. They are most marked at the end sections and are probably due to warping produced by the end constraints. In one case only do they become important,—*i.e.*, at the end of the double-angle specimen (Figs. 19 and 21). For this section the mean straight lines could not be drawn and the results have not been reduced. These deviations, however, entirely vanish at the central section of the same specimen, and are thus probably due to the great end constraints in this case.

[10] *Loc. cit.*

§2. *The Method of Analysis.*

The planar distribution of stress having been established, the equations of Part I may be applied to determine the position of the load axis at each section. The details of this analysis are given in Table IV. The method, being the same for all cases, may be illustrated by a single example. Consider the central section of Specimen I with lock angle at 20,000 pounds load. The distribution of strain is shown in Fig. 8, and the constants for the section are given at the head of Table I. The results of the

TABLE I.

Stresses Corresponding to the Mean Extensometer Readings for Specimen I.

Constants.—Area of cross-section, 1.60 square inches.
Distance from centroid to back of angle, 0.85 inch.
$I = 1.37$ (inch)^4. $J = -0.81$ (inch)^4.
$E = 29.1 \times 10^6$ pounds per square inch.
Stresses in pounds per square inch. Tension +. Compression −.

	Load N. (lbs.)	1	2	3	4	5	6	7	8	9	10
(a) *With lock angle.*											
Central section	5,000	− 2,380	− 360	1,450	2,980	5,020	6,180	5,960	5,520	5,310	5,310
	10,000	− 3,420	− 440	3,060	6,040	9,810	12,140	11,560	10,980	10,450	10,310
	15,000	− 5,020	− 360	4,510	8,940	14,390	17,720	17,000	16,300	15,540	15,280
	20,000	− 5,960	− 70	6,250	11,910	18,810	23,280	22,400	21,440	20,580	20,280
Section 2½″ from end plate	5,000	− 2,470	− 510	1,310	2,620	4,220	6,400	6,250	6,110	6,110	6,110
	10,000	− 4,870	− 940	2,480	5,380	8,500	12,700	12,420	12,140	12,140	12,070
	15,000	− 7,060	−1,240	3,780	8,290	13,090	19,010	18,450	18,170	18,100	18,100
	20,000	− 9,090	−1,450	5,240	11,200	17,380	25,210	24,350	24,000	23,700	23,400
(b) *Without lock angle.*											
Central section	5,000	− 2,030	− 580	1,380	2,690	4,290	5,750	5,680	5,600	5,530	5,390
	10,000	− 3,640	− 730	2,760	5,390	8,440	11,490	11,490	11,330	11,200	10,830
	15,000	− 4,940	− 800	4,290	8,000	12,420	17,000	17,000	16,880	16,800	16,380
	20,000	− 6,030	− 730	5,810	10,690	16,420	22,470	22,400	22,240	22,240	21,780
Section 2½″ from end plate	5,000	− 2,690	− 870	650	2,260	4,290	5,750	5,670	5,600	5,520	5,380
	10,000	− 5,390	−1,960	1,530	4,510	8,440	12,160	12,220	12,380	12,720	12,720
	15,000	− 7,930	−2,620	2,470	6,840	12,420	18,180	18,270	18,320	18,920	18,990
	20,000	−10,100	−3,270	3,640	9,240	16,420	24,210	24,280	24,210	24,850	25,100

calculation are given in the first line of Table IV. Lines through the centroid of the section parallel to the legs of the angle being taken as coördinate axes, the point of zero stress in the angle is seen to have coördinates $(1.15, -0.85)$. This is one point on the neutral axis. Another may be found by calculating where the 6−10 line would cut the x-axis if produced. Thus the neutral axis is found to be inclined to the x-axis at an angle of $\tan^{-1} - 6.93$. This is $\tan\alpha$ in the notation of Part I. Substituting this value, the

coördinates of the point of zero stress and $f = o$ in equations (2) and (3), the coördinates (x_k, y_k) of the load axis are found to be $(-0.76, 0.37)$. All the values of (x_k, y_k) given in Table IV were calculated in a similar manner. The coördinates (x_k, y_k) give the position of the load axis relative to the cross-section considered. This depends, to some extent, upon the lateral deflection

TABLE II.

Stresses Corresponding to the Mean Extensometer Readings for Specimen II.

Constants.—Area of cross-section, 1.49 square inches.
Distance from centroid to back of angle, 0.85 inch.
$I = 1.29$ (inch)^4. $J = -0.76$ (inch).^{4}
$E = 31.1 \times 10^6$ pounds per square inch.
Stresses in pounds per square inch. Tension +. Compression −.

	Load N. (lbs.)	1	2	3	4	5	6	7	8	9	10
(a) *With lock angle.*											
Central section	5,000	− 2,100	− 310	1,250	3,040	4,590	6,300	6,230	5,990	5,990	5,920
	10,000	− 3,890	− 310	2,650	5,920	9,180	12,380	12,300	12,070	12,070	11,980
	15,000	− 5,130	− 150	4,200	8,950	13,610	18,350	18,290	18,120	17,980	17,810
	20,000	− 6,850	+ 540	5,760	11,980	18,130	24,200	24,150	23,900	23,810	23,500
Section 2½" from end plate	5,000	− 2,340	− 780	780	2,490	4,050	6,150	6,150	6,300	6,300	6,780
	10,000	− 4,510	−1,400	1,870	5,370	8,480	12,450	12,370	12,510	12,770	13,210
	15,000	− 6,620	−1,870	2,960	8,250	12,830	18,750	18,500	18,750	18,990	19,600
	20,000	− 8,560	−2,100	4,440	11,280	17,280	24,900	24,650	24,750	24,970	25,680
(b) *Without lock angle.*											
Central section	5,000	− 2,250	− 700	930	2,410	4,040	6,230	6,150	6,230	6,230	6,230
	10,000	− 4,050	−1,090	2,020	5,140	8,100	12,210	12,210	12,380	12,430	12,430
	15,000	− 5,600	−1,320	3,270	7,860	12,100	18,220	18,290	18,430	18,600	18,680
	20,000	− 6,920	−1,320	4,590	10,500	15,880	23,800	24,100	24,280	24,680	24,820
Section 2½" from end plate	5,000	− 2,410	− 860	700	2,410	3,970	6,150	6,230	6,610	6,840	7,000
	10,000	− 4,590	−1,550	1,550	5,060	8,160	12,380	12,780	13,080	13,540	14,220
	15,000	− 6,620	−2,100	2,570	7,780	12,420	18,600	18,980	19,600	20,500	21,170
	20,000	− 8,490	−2,410	3,660	10,420	16,720	24,600	25,100	25,900	26,440	27,700
(c) *Load axis on gusset plate changed to line of rivets.*											
Central section	5,000	− 2,880	− 780	1,010	2,490	4,360	6,300	6,300	6,300	6,300	6,300
	10,000	− 5,290	−1,400	2,260	5,290	8,640	12,450	12,480	12,480	12,680	12,480
	15,000	− 7,160	−1,630	3,660	7,860	12,820	18,290	18,420	18,590	18,810	18,810
	20,000	− 8,710	−1,550	5,130	10,670	16,950	23,800	24,250	24,420	24,800	24,980
Section 2½" from end plate	5,000	− 3,190	−1,170	620	2,410	4,510	6,460	6,460	6,530	6,530	6,850
	10,000	− 6,230	−2,180	1,480	5,060	9,100	12,900	12,730	12,900	13,130	13,600
	15,000	− 8,950	−2,880	2,640	7,780	13,520	19,120	19,050	19,210	19,520	20,300
	20,000	−11,280	−3,500	3,680	10,420	18,280	25,200	25,120	25,280	25,750	26,420

of the section, so that, in order to find the position of the load axis where the load enters the angle, it is necessary to correct for this deflection. Column (4) of Table IV gives the mean deflections of the centroids of the sections under the different loads measured as described above, and column (5) gives the values of (x_k, y_k) referred to the end sections of the specimens taken as being the middle section of the riveted ends.

The value of E for each specimen may also be found from the experimental results. In the case considered above the ratio of maximum to mean stress for the calculated position of the load axis computed from equation (2) is 1.945 [11] The elongation due

TABLE III.

Stresses Corresponding to the Mean Extensometer Readings for Specimen III (Double-angle).

Constants (Each angle).—Area of cross-section, 1.54 square inches.
Distance from centroid to back of angle, 0.85 inch.
$I = 1.33$ (inch).[4] $J = -0.78$ (inch).[4]
$E = 30.1 \times 10^6$ pounds per square inch.
Stresses in pounds per square inch. Tension +. Compression −.

	Load N. (lbs.)	1	2	3	4	5	6	7	8	9	10
(a) *With lock angle.*											
Central section, left.	15,000	5,040	5,190	5,190	5,110	5,190	4,960	4,890	4,810	4,590	4,210
	20,000	6,700	6,920	6,850	6,850	7,070	6,620	6,550	6,400	6,250	5,710
	25,000	8,440	8,660	8,580	8,500	8,660	8,360	8,130	7,980	7,830	7,220
	30,000	10,080	10,300	10,170	10,170	10,470	9,930	9,860	9,640	9,410	8,730
Central section, right.	15,000	4,800	5,040	5,190	5,270	5,490	5,190	4,820	4,510	4,060	3,610
	20,000	6,630	6,700	6,930	7,080	7,310	7,000	6,470	6,160	5,560	4890
	25,000	8,210	8,290	8,660	8,960	9,260	8,650	8,130	7,750	6,920	6,240
	30,000	9,800	9,940	10,380	10,690	10,990	10,380	9,770	9,250	8,350	7,520
Section 2¼″ from end plate, left.	15,000	2,860	3,690	4,580	5,410	5,940	6,310	6,310	5,650	5,560	5,260
	20,000	3,760	5,110	6,090	7,300	7,680	8,280	7,900	7,670	7,230	7,070
	25,000	4,890	6,390	7,680	9,030	9,710	10,390	9,930	9,400	9,250	8,800
	30,000	5,940	7,600	9,250	10,900	11,580	12,490	11,970	11,370	10,500	10,600
Section 2¼″ from end plate, right.	15,000	2,635	3,610	4,280	4,960	5,340	5,710	5,860	5,490	5,860	5,560
	20,000	3,760	4,890	5,860	6,690	7,300	7,150	7,450	7,220	7,370	7,300
	25,000	4,740	6,240	7,450	8,270	8,950	9,410	9,180	9,110	9,180	9,260
	30,000	5,860	7,520	8,960	10,000	10,680	11,340	11,030	10,890	11,200	11,030
(b) *Without lock angle.*											
Central section, left.	15,000	4,220	4,360	4,660	4,890	5,190	5,340	5,410	5,260	5,190	4,960
	20,000	5,650	5,850	6,320	6,470	7,000	7,070	7,150	7,070	7,000	6,760
	25,000	7,070	7,380	7,830	8,050	8,650	8,880	8,950	8,880	8,800	8,500
	30,000	8,430	8,880	9,410	9,860	10,390	10,600	10,680	10,600	10,530	10,380
Central section, right.	15,000	3,900	4,510	4,890	5,260	5,790	5,650	5,490	5,120	4,820	4,440
	20,000	5,340	6,020	6,550	6,920	8,280	7,670	7,230	6,920	6,400	6,020
	25,000	6,780	7,450	8,200	8,730	9,480	9,480	9,180	8,580	8,050	7,520
	30,000	8,130	8,950	9,790	10,430	11,410	11,420	10,990	10,220	9,860	9,180
Section 2¼″ from end plate, left.	15,000	3,080	4,280	5,420	6,170	6,620	6,320	5,640	4,960	4,580	3,980
	20,000	4,290	5,720	7,150	8,280	8,730	8,420	7,680	6,770	6,090	5,340
	25,000	5,340	7,220	9,180	10,300	10,990	10,460	9,470	8,570	7,670	5,690
	30,000	6,620	8,740	10,900	12,490	13,220	12,500	11,500	10,150	9,250	7,960
Section 2¼″ from end plate, right.	15,000	2,860	3,990	5,110	5,800	6,250	5,950	5,560	4,960	4,660	4,220
	20,000	3,990	5,490	6,850	7,900	8,280	7,900	7,370	6,620	6,170	5,650
	25,000	4,970	6,920	8,650	9,780	10,380	10,000	9,260	8,350	7,670	7,070
	30,000	6,100	8,280	10,440	11,890	12,490	12,030	11,200	10,090	9,180	8,500

[11] It would, of course, be incorrect to take the mean height of the plotted diagrams for a given section as being the mean extension over the section, since the curves only give the distribution of strain over the outside faces of the angles.

TABLE IV.

Reduction of Experimental Results.

Specimen and section	Load (lbs.)	1	2	3		4		5		6	7
		Inclination of neutral line	Point of zero stress $y = -0.85''$	Load axis referred to axes through centroid of section		Lateral deflection of centroid of section referred to mid rivet section		Load axis referred to axes through mid rivet section		Max. stress lbs. per sq. in.	Ratio of maximum to mean stress
		$\tan\alpha$	x	x_h	y_h	x	y	x_h	y_h		
		(all measured in inches)									
Specimen I. With lock angle.											
				−	+			−	+		
Central....	5,000	− 6.11	1.05	0.85	0.40	0.04	0.01	0.89	0.41	6,480	2.08
	10,000	− 5.73	1.10	0.81	0.37	0.08	0.02	0.89	0.40	12,720	2.05
	15,000	− 6.40	1.11	0.79	0.38	0.11	0.02	0.90	0.40	18,550	2.00
	20,000	− 6.93	1.15	0.76	0.37	0.15	0.03	0.91	0.40	24.080	1.94
					Mean values			0.90	0.40		
				−	+			−	+		
End.....	5,000	−15.00	0.95	0.92	0.50	0.03	0.00	0.95	0.50	6,550	2.04
	10,000	−20.95	0.95	0.92	0.51	0.05	0.01	0.97	0.52	12,720	1.98
	15,000	−16.26	0.97	0.90	0.49	0.07	0.01	0.97	0.50	19,420	1.98
	20,000	−14.81	0.97	0.90	0.49	0.09	0.02	0.99	0.51	25,540	2.00
					Mean values			0.97	0.51		
Specimen I. Without lock angle.											
				−	+			−	+		
Central....	5,000	−16.5	0.99	0.88	0.48	0.04	0.01	0.92	0.49	5,900	1.96
	10,000	−31.6	1.05	0.82	0.47	0.08	0.01	0.90	0.48	11,640	1.86
	15,000	−28.4	1.10	0.78	0.44	0.12	0.01	0.90	0.46	18,030	1.82
	20,000	−52.9	1.11	0.77	0.45	0.15	0.02	0.92	0.47	22,620	1.79
					Mean values			0.91	0.47		
				−	+			−	+		
End.....	5,000	24.30	0.87	0.97	0.60	0.03	0.00	1.00	0.60	6,480	2.04
	10,000	19.15	0.87	0.97	0.60	0.05	0.00	1.02	0.61	13,100	2.05
	15,000	15.58	0.87	0.96	0.61	0.07	0.01	1.03	0.62	19,500	2.06
	20,000	14.45	0.88	0.95	0.60	0.09	0.01	1.04	0.61	25,820	2.07
					Mean values			1.02	0.61		
Specimen II. With lock angle.											
				−	+			−	+		
Central....	5,000	−16.4	1.00	0.88	0.48	0.03	0.00	0.91	0.48	6,300	1.95
	10,000	−41.6	1.07	0.81	0.46	0.07	0.00	0.88	0.46	12,450	1.83
	15,000	−30.7	1.10	0.79	0.45	0.10	0.01	0.89	0.46	18,670	1.82
	20,000	−59.2	1.12	0.78	0.45	0.13	0.01	0.91	0.46	24,260	1.78
					Mean values			0.90	0.46		
				−	+			−	+		
End.....	5,000	10.6	0.90	0.93	0.61	0.02	0.00	0.95	0.60	6,920	2.08
	10,000	20.4	0.91	0.93	0.58	0.04	0.00	0.97	0.58	13,520	2.00
	15,000	23.0	0.95	0.90	0.55	0.06	0.01	0.96	0.56	19,820	1.96
	20,000	42.7	0.98	0.88	0.53	0.08	0.01	0.96	0.54	26,150	1.90
					Mean values			0.96	0.57		
Specimen II. Without lock angle.											
				−	+			−	+		
Central....	5,000	∞	0.90	0.96	0.57	0.04	0.00	1.00	0.57	6,220	1.94
	10,000	87.60	0.93	0.93	0.55	0.08	0.00	1.01	0.55	12,370	1.93
	15,000	42.20	0.98	0.88	0.53	0.12	0.01	1.00	0.54	18,650	1.90
	20,000	23.30	1.00	0.86	0.53	0.14	0.01	1.00	0.54	25,100	1.83
					Mean values			1.00	0.55		
				−	+			−	+		
End.....	5,000	7.01	0.86	0.96	0.65	0.03	0.00	0.99	0.65	7,380	2.18
	10,000	8.20	0.91	0.92	0.61	0.04	0.00	0.96	0.61	14,470	2.09
	15,000	7.94	0.92	0.91	0.60	0.06	0.01	0.97	0.61	21,600	2.09
	20,000	9.40	0.96	0.88	0.57	0.08	0.01	0.96	0.58	28,000	2.03
					Mean values			0.97	0.61		

TABLE IV—*Continued.*

Reduction of Experimental Results.

Specimen and section	Load (lbs.)	1	2	3		4		5		6	7
		Inclination of neutral line	Point of zero stress $y = -6.85''$	Load axis referred to axes through centroid of section		Lateral deflection of centroid of section referred to mid rivet section		Load axis referred to axes through mid rivet section		Max. stress lbs. per sq. in.	Ratio of maximum to mean stress
		$\tan\alpha$	x	x_1	y_1	x	y	x_1	y_1		
		(all measured in inches)									
Specimen II. Load axis changed to line of rivets.											
Central	5,000	∞	0.90	−0.96	+0.57	0.04	0.00	−1.00	+0.57	6,300	1.94
	10,000	53.30	0.95	0.91	0.55	0.09	0.00	1.00	0.54	12,820	1.92
	15,000	29.20	0.99	0.87	0.53	0.12	0.01	0.99	0.54	19,110	1.90
	20,000	27.35	1.02	0.84	0.52	0.15	0.01	0.99	0.52	25,260	1.89
						Mean values		1.00	0.54		
End	5,000	30.00	0.81	−1.05	+0.64	0.03	0.00	−1.08	+0.41	6,840	2.11
	10,000	58.80	0.83	1.03	0.62	0.05	0.00	1.08	0.40	13,220	2.05
	15,000	38.60	0.87	0.99	0.60	0.07	0.00	1.07	0.41	19,800	2.02
	20,000	111.30	0.90	0.96	0.57	0.10	0.00	1.06	0.40	25,740	1.96
						Mean values		1.07	0.40		
Specimen III. Double-angle with lock angles.											
Left angle central	15,000	− 0.15	115.0	+0.02	−0.05	Lateral deflections negligible				5,270	1.06
	20,000	− 0.12	153.3	0.02	0.05					6,960	1.06
	25,000	− 0.14	142.8	0.02	0.04					8,650	1.05
	30,000	− 0.15	136.6	0.02	0.04					10,340	1.05
		Mean values		0.02	0.04						
Left angle end	15,000	Non-planar distribution of stress								6,240	1.26
	20,000									8,320	1.26
	25,000									10,700	1.30
	30,000									12,720	1.29
Right angle central	15,000	− 0.39	17.90	+0.03	−0.11	Lateral deflections negligible				5,640	1.19
	20,000	− 0.36	19.15	0.03	0.11					7,520	1.19
	25,000	− 0.39	18.66	0.03	0.11					9,410	1.18
	30,000	− 0.40	18.75	0.02	0.10					11,200	1.18
		Mean values		0.03	0.11						
Right angle end	15,000	Non-planar distribution of stress								5,640	1.19
	20,000									7,520	1.19
	25,000									9,410	1.19
	30,000									11,200	1.14
Specimen III. Double-angle without lock angles.											
Left angle central	15,000	− 4.20	9.51	−0.08	+0.03	Lateral deflections negligible				5,460	1.11
	20,000	− 4.03	9.52	0.08	0.03					7,330	1.11
	25,000	− 6.61	10.05	0.08	0.04					9,030	1.10
	30,000	− 7.20	10.52	0.08	0.04					10,750	1.09
		Mean values		0.08	0.03						
Left angle end	15,000	Non-planar distribution of stress								6,810	1.39
	20,000									9,190	1.40
	25,000									11,360	1.38
	30,000									13,530	1.38
Right angle central	15,000	− 1.34	7.39	−0.07	−0.02	Lateral deflections negligible				5,950	1.22
	20,000	− 1.38	6.83	0.08	0.02					7,980	1.24
	25,000	− 1.33	7.13	0.07	0.02					9,970	1.23
	30,000	− 1.35	7.18	0.07	0.02					12,000	1.23
		Mean values		0.07	0.02						
Right angle end	15,000	Non-planar distribution of stress								6,440	1.32
	20,000									8,510	1.32
	25,000									10,710	1.33
	30,000									12,980	1.33

to the maximum stress is, from the curve in Fig. 8, 0.00331 inch. Thus the mean elongation over a length of 4 inches for a load of 20,000 pounds is $\frac{0.00331}{1.945} = 0.0017$ inch. This corresponds to a value of E given by $E = \frac{20000}{1.60} \times \frac{4}{0.0017} = 29.4 \times 10^6$ pounds per square inch. Proceeding in this manner, values of E were found for all the loadings at the central sections of the specimens, and the mean results are given at the heads of Tables I, II and III. The variations of E found in this way may be taken as a measure of the maximum over-all error in observation, measurement of the loads, calibration of the extensometers, plotting and calculation, and due to variations in the material. For Specimen I the maximum variation was 5 per cent., but this was in two cases only, the rest of the readings giving results which did not differ more than 2 per cent. The results for Specimen II were similar, while for the double angle, taking a mean of both angles, the deviation was in no case greater than 1 per cent.

§3. *Discussion of the Results—General.*

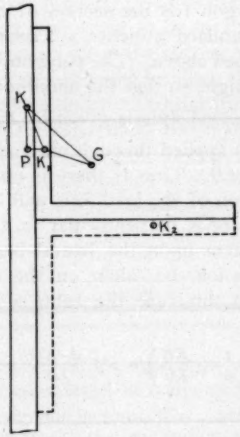
If the load axis remained unchanged relatively to the section as the load increased, all the mean straight lines for any particular section would pass through the same points and the position of the neutral axis would be independent of the load. An inspection of the curves, Figs. 8 to 21, will show that this is not the case, and the discrepancy is due to the alteration of the load axis owing to lateral bending of the specimen. When this is allowed for by referring all load axes to the mean section through the end connections, the corrected axis remains practically constant as the load increases, as may be seen from the results given in Table IV, column 5. This is especially noticeable at the central sections. At the end sections there is in every case a slight shift, as the load increases, away from the centroid of the section perpendicular to the end plate and towards the centroid parallel to the end plate. This is probably due to a change in the fixing couple at the ends as the load increases.

§4. *The Effect of Constraints.*

In the case of single angles having long, narrow end plates the experimentally-determined load axis at the central section corrected for deflection never deviated more than 0.02 inch from the

line of rivets. In the present tests y_k varies from 0.40 inch to 0.57 inch, while the y coördinate of the line of rivets is 0.90 inch. Thus the deviation from the line of rivets ranges from 0.33 inch to 0.50 inch. The reason for this is that the end plates in the present case are very much stiffer in their own plane than those used in the earlier experiments. The effect of such end constraints upon the position of the load axis and on the ratio of maximum to mean stress will first be considered in general. Let K in Fig. 22

FIG. 22.



represent the position of the load axis when there are no end constraints and K_1 its position as altered due to partial fixing. Assuming a normal load N, the bending couple in the former case would be $N.KG$ and in the latter $N.K_1G$, the restraining couple being thus $N.KK_1$. This couple may be resolved into the components $N.KP$ and $N.K_1P$ respectively parallel and perpendicular to the end plate. If the end connection consists of a gusset plate only, the former couple is the restraint due to its stiffness against bending in its own plane and the latter that due to its stiffness against bending perpendicular to its plane. It will readily be seen

that the former must be much greater than the latter for an ordinary thin plate, and thus KK_1 will be nearly parallel to the connected leg. A lock angle may act in two ways. It may restrain the bending more or less and it may transfer part of the load to the line of rivets by which it is connected to the main angle (K_2). The latter would cause a movement of the line of pull along KK_2, and the former a somewhat similar motion, so that the two effects cannot be separated.

The effect of these changes of axis upon the ratio of maximum to mean stress may be studied by means of the S-polygon. Fig. 2 represents this polygon for the section of Specimen I (slightly heavier than the standard 3 inches × 3 inches × ¼ inch angle) calculated as described above. The polygon is drawn to the same linear scale as the angle, so that the maximum bending stress for a position K of the load axis is given by $N \cdot \frac{KG}{LG}$ as described in Part I. The load is applied through one row of rivets 1.75 inch from B, in the leg AB. Thus if there is no restraint due to the end plate, the position of the load axis will be approximately K. (The exact position of K perpendicular to the connected leg will depend to some extent upon the lateral bending, but may, for purposes of illustration, be taken on the outside face of the connected leg.) In this case the ratio of maximum to mean stress will be

$$A\left(\frac{1}{A}+\frac{KG}{LG}\right)=1+\frac{A\cdot KG}{LG}=2.82,$$

and the maximum stress will occur at the corner A. If, however, a constraining couple be introduced parallel to AB, the point of loading will move from K along AB toward B. As the couple increases the ratio $\frac{KG}{LG}$ will obviously decrease until the point K_1 on the line joining the apex (ab) to G is reached. Thus the restraining couple will obviously decrease the ratio of maximum to mean stress. At the point K_1 this ratio is 1.89, a decrease of 33 per cent., and the stress is constant over AB. From K_1 to B the ratio $\frac{KG}{LG}$ again increases, and thus the ratio of maximum to mean stress increases. At the point K_2, level with the centroid of the angle, this ratio is 3.18, greater than without constraint. Thus

the most favorable position of the load axis is on the line joining the apex (ab) to the centroid. From this it will be seen that an increase of constraint is not always an advantage.

With the load axis in any of the positions considered it will be noticed that a constraint perpendicular to the end plate, causing K to move inwards, always leads to a decrease of $\frac{KG}{LG}$, *i.e.*, a decrease in the ratio of maximum to mean stress. The only effective way of obtaining such restraint is by having another angle back to back with the first, *i.e.*, a double-angle section. In this case, as will be seen later, there is a very considerable restraining couple parallel to BC, causing the stress to be much more evenly distributed over the section. This is the correct way of considering such a section, and not as one piece bending about a neutral axis parallel to BC, as is often done. These points will be discussed when the experimental results upon the double angle are considered.

The effects of constraints upon the distribution of stress in other types of section, such as unequal-legged angles, Z-sections, etc., may be considered in a similar manner by use of the S-polygon.

§5. *Experimental Results.* (a) *The Restraining Effect of the End Plates.*

The results of the tests on the specimens with the lock angles removed will first be considered in order to find the restraining effects due solely to the end plate. The restraining couple perpendicular to the end plate,—*i.e.*, in the direction Ox,—is difficult to ascertain exactly, because it is neither certain where the load enters the angle nor exactly what is the position of the load axis through the thickness of the end plate. In Table IV, column 5, the position of x_k relative to the mid-section of the rivets is given, and it may be remarked that it remains practically constant as the load increases in each of the Specimens I and II, although it is not the same for each, the mean being $x_k = -0.91$ for Specimen I and $x_k = -1.00$ for Specimen II. In any case, however, the effect of this restraint is small and the specimen may be considered as practically free to bend perpendicularly to the end

plate. This result is in agreement with the earlier experiments made with narrow end plates. In these it was found that $x_k = -0.91$ both with an end plate ¼ inch thick and with one ¾ inch thick.

The case of the double angle is quite different. The two angles, back to back, prevent each other from bending perpendicularly to the end plate, and the result is an almost complete fixing in that direction, considerably decreasing the ratio of maximum to mean stress over the section. Again, the effect is practically independent of the load and is, as might be expected, the same for each angle. The arms of the fixing couples parallel to Ox, measured from the back of the main angles, are given in the annexed table.

TABLE V.

Load	Left angle	Right angle
15,000.....	0.77	0.77
20,000.....	0.77	0.76
25,000.....	0.77	0.77
30,000.....	0.77	0.76
Mean.....	0.77	0.77

The fixing parallel to the end plate is much more important for the single angles and of equal importance for the double angle. The annexed table shows the distance, measured parallel to Oy, of the actual axis from the line of rivets for each of the specimens at the central section:

TABLE VI.

Load, pounds	Specimen I, inch	Specimen II, inch	Double angle		
			Load, pounds	Left, inch	Right, inch
5,000...	0.41	0.33	15,000...	0.87	0.88
10,000...	0.42	0.35	20,000...	0.87	0.88
15,000...	0.44	0.36	25,000...	0.86	0.88
20,000...	0.43	0.36	30,000...	0.86	0.88
Mean...	0.42	0.35	Mean...	0.86	0.88

It will be noticed that there is fair agreement between Specimen I and Specimen II, although the line of pull on the end

plate is different in the two cases. The fixing moment of the double angle, however, is more than twice that of the single angle.

In the earlier experiments on single angles connected to long, narrow gusset plates the line of pull, as mentioned above, never deviated more than 0.02 inch from the line of rivets. This shows the important effect of the type of gusset plate upon the position of the load axis and hence on the maximum stress in the member. The latter, of course, depends upon the lateral bending of the specimen, and in calculating the figures in column 7, Table IV, the axis of loading was not corrected for deflection. It will be noticed that, with a few exceptions, the ratio at the central sections decreases as the load increases. This is due to the alteration of the axis due to lateral bending. In every case the ratio is much less than for a load coinciding with the line of rivets, showing the value of a wide and firmly-connected gusset plate. The ratios of maximum to mean stress for a line of pull having the same x_k as that given by the experimental results, but y_k in line with the rivets, have been found by means of the S-polygon, as described above, and are compared with the actual ratios in the annexed table.

TABLE VII.

Load, pounds	Specimen I			Specimen II		
	Actual	Line of pull in line of rivets	Decrease due to fixing	Actual	Line of pull in line of rivets	Decrease due to fixing
			Per cent.			*Per cent.*
5,000.....	1.96	2.62	25.2	1.94	2.80	30.7
10,000.....	1.86	2.66	30.0	1.93	2.83	31.8
15,000.....	1.82	2.69	32.4	1.90	2.86	33.6
20,000.....	1.79	2.70	33.7	1.83	2.88	36.5

The average decrease of the ratio of maximum to mean stress due to the stiffness of the end plate in its own plane is thus about 35 per cent. at the highest load, the load axis (K^1, Fig. 2) being brought almost into its most favorable position as described in the last section.

At the end sections the ratio of maximum to mean stress is always greater than at the centre, the percentage increase being as shown in the annexed table.

TABLE VIII.

The Ratio of Maximum to Mean Stress at the End Section Compared with that at the Central Section.

Load, pounds	Specimen I			Specimen II		
	End	Central	Per cent. increase	End	Central	Per cent. increase
5,000.....	2.04	1.96	4.1	2.18	1.94	12.4
10,000.....	2.05	1.86	10.2	2.09	1.93	8.3
15,000.....	2.06	1.82	13.2	2.09	1.90	10.0
20,000.....	2.07	1.79	15.6	2.03	1.83	10.9
	Double angle					
	Left			Right		
	End	Central	Per cent. increase	End	Central	Per cent. increase
15,000.....	1.39	1.11	25.2	1.32	1.22	8.2
20,000.....	1.40	1.11	26.1	1.32	1.24	6.5
25,000.....	1.38	1.10	25.5	1.33	1.23	8.1
30,000.....	1.38	1.09	26.6	1.33	1.23	8.1

It will be noticed that the percentage increase varies widely, as might be expected. Tertiary stresses are indicated, ranging from 4 per cent. to 15 per cent. for the single angles and rising as high as 26 per cent. for the double angles. The curves for the latter show that the distribution of stress departed greatly from the planar. There is probably a readjustment of the longitudinal variations of stress at the different loads, depending upon the end connections, so that the results are not so reliable as at the central section and the exact tertiary stresses are thus indeterminate. They are local, however, and thus not so important, and may probably be neglected in designing if ordinary working stresses are used and the correct variation of stress over the main part of the member considered.

§5. (b) *The Effect of Lock Angles.*

It is often claimed, as stated in the introduction, that lock angles considerably reduce the ratio of maximum to mean stress in the member, some authorities going even so far as to say that when a lock angle is used the stress is practically uniformly distributed. A glance at the adjoined table will show how far this is from being the truth.

TABLE IX.

The Effect of Lock Angles on the Ratio of Maximum to Mean Stress at the Central Cross-sections.

Load, pounds	Specimen I			Specimen II		
	Ratio of maximum to mean stress		Per cent. change due to lock	Ratio of maximum to mean stress		Per cent. change due to lock
	With lock	Without lock		With lock	Without lock	
			Increase			*Decrease*
5,000.....	2.08	1.96	6.1	1.95	1.95	0
10,000.....	2.05	1.86	10.2	1.83	1.93	5.2
15,000.....	2.00	1.82	9.9	1.82	1.90	4.2
20,000.....	1.94	1.79	8.4	1.78	1.83	2.7
	Double Angle, left.			Double angle, right.		
			Decrease			*Decrease*
15,000.....	1.06	1.11	4.5	1.19	1.22	2.5
20,000.....	1.06	1.11	4.5	1.19	1.24	4.0
25,000.....	1.05	1.10	4.5	1.18	1.23	3.2
30,000.....	1.05	1.10	4.5	1.18	1.23	3.2

This table compares the ratio of maximum to mean stress with the lock angles in position and with them removed. Considering the single angles first, it will be seen that in the case of Specimen II the ratio of maximum to mean stress is decreased about 3 per cent. at working load by the use of the lock angle, while in Specimen I it is actually greater by about 8.4 per cent. with the lock angle than without it. In the case of the double angle the stress in one leg is decreased about 4.5 per cent. and in the other 3.2 per cent. It is thus evident that the effect of the lock angle is very small, and that it is practically worthless for the purpose of distributing the stress more uniformly.

The variability, and apparently paradoxical character of these results will be explained by a consideration of the shift of the load axis due to the lock angle as displayed by the adjoined table.

It will be seen that in all cases, as might be expected, the lock angle slightly increases the arms of the restraining couples both perpendicular to and parallel to the end plate. The change of y_k ranges from 0.07 inch to 0.09 inch, while that of x_k is 0.10 inch in all cases except Specimen I, in which it is only 0.01 inch. The agreement between the results in the different cases is remarkable, especially considering that no care was taken to get uniform work-

TABLE X.

Change of Load Axis at Central Section Due to Lock Angle.

Load, pounds	x_k		Shift toward centroid produced by lock, inches	y_k		Shift toward centroid produced by lock, inches
	With lock, inches	Without lock, inches		With lock, inches	Without lock inches	
	Specimen I					
	−	−		+	+	
5,000.....	0.89	0.92	0.03	0.41	0.49	0.08
10,000.....	0.89	0.90	0.01	0.40	0.48	0.08
15,000.....	0.90	0.90	0.00	0.40	0.46	0.06
20,000.....	0.91	0.92	0.01	0.40	0.47	0.07
	Mean displacement...		0.01	Mean displacement.		0.07
	Specimen II					
	−	−		+	+	
5,000.....	0.91	1.00	0.09	0.48	0.57	0.09
10,000.....	0.88	1.01	0.13	0.46	0.55	0.09
15,000.....	0.89	1.00	0.11	0.46	0.54	0.08
20,000.....	0.91	1.00	0.09	0.46	0.54	0.08
	Mean displacement...		0.10	Mean displacement.		0.08
	Double angle Left					
	+	−		−	+	
15,000.....	0.02	0.08	0.10	0.05	0.03	0.08
20,000.....	0.02	0.08	0.10	0.05	0.03	0.08
25,000.....	0.02	0.08	0.10	0.04	0.04	0.08
30,000.....	0.02	0.08	0.10	0.04	0.04	0.08
	Mean displacement...		0.10	Mean displacement.		0.08
	Double angle Right					
	+	−		−	−	
15,000.....	0.03	0.07	0.10	0.11	0.02	0.09
20,000.....	0.03	0.08	0.11	0.11	0.02	0.09
25,000.....	0.03	0.07	0.10	0.11	0.02	0.09
30,000.....	0.02	0.07	0.09	0.10	0.02	0.08
	Mean displacement...		0.10	Mean displacement.		0.09

manship in the making of the specimens. It may be said that the lock angle produces an increase of, roughly, $^1/_{10}$ inch in the arms of each of the restraining couples. Now the effect of this depends upon the position of the load axis due to the stiffness of the end plate. In Specimen I the displacement of the axis brings the *GL* line of the S-polygon from *GL'* to *GL''* (Fig. 2). *L* thus passes over from the *a* line to the *b* line, and the maximum stress is actu-

ally increased by the additional restraint. In the other specimens the point L remains on the a line, and thus the restraint slightly diminishes the maximum stress.

The effect of the lock angle is no more marked at the end sections than at the central section, the change in the restraining arm due to its action varying from 0.02 inch to 0.10 inch.

TABLE XI.

The Effect of Lock Angles on the Ratio of Maximum to Mean Stress. End Sections.

Load, pounds	Specimen I Ratio of maximum to mean stress			Specimen II Ratio of maximum to mean stress		
	With lock	Without lock	Decrease due to lock	With lock	Without lock	Decrease due to lock
			Per cent.			*Per cent.*
5,000.....	2.04	2.04	0.0	2.08	2.18	4.6
10,000.....	1.98	2.05	3.4	2.00	2.09	4.3
15,000.....	1.98	2.06	3.9	1.96	2.09	6.2
20,000.....	2.00	2.07	3.4	1.90	2.03	6.4
	Double angle					
	Left			Right		
15,000.....	1.26	1.39	9.3	1.19	1.32	9.9
20,000.....	1.26	1.40	10.0	1.19	1.32	9.9
25,000.....	1.30	1.38	5.8	1.19	1.33	10.5
30,000.....	1.29	1.38	6.5	1.14	1.33	14.3

TABLE XII.

Change of Load Axis Due to Lock Angles. End Sections.

Load, pounds	x_h (inches)			y_h (inches)		
	With lock	Without lock	Change due to lock	With lock	Without lock	change due to lock
Specimen I						
5,000...	0.95	1.00	0.05	0.51	0.61	0.10
10,000...	0.97	1.02	0.05	0.50	0.62	0.12
15,000...	0.97	1.03	0.06	0.50	0.61	0.09
20,000...	0.97	1.04	0.05	0.51	0.60	0.09
			0.05			0.10
Specimen II						
5,000...	0.95	0.99	0.04	0.60	0.65	0.05
10,000...	0.91	0.96	0.05	0.58	0.61	0.03
15,000...	0.96	0.97	0.01	0.56	0.61	0.03
20,000...	0.96	0.96	0.00	0.54	0.58	0.04
			0.02			0.04

It produces a slight decrease in the maximum stress, 3 per cent. to 7 per cent., for the single angles and a little greater, 6 per cent. to 10 per cent. and in one case 14 per cent., in the double angles. The results are more variable than at the centre, owing, no doubt, to the difference in the fixing couples near the rivets in the different specimens due to the different positions of the line of loading of the gusset plate.

To sum up, it may be said that neither in the single nor double angles does the lock angle cause a displacement of more than $^1/_{10}$ inch in the line of pull either parallel or perpendicular to the end plate, and that this produces only small changes in the maximum stress, insignificant in comparison with those arising from the stiffness of the gusset plate in its own plane. The effect of the lock angle is thus so small as to be practically negligible. It is also uncertain, since, depending upon the stiffness of the gusset plate, which is difficult to predict, it may increase or diminish the maximum stress. The contention that the lock angle increases the virtual length of attachment of the main angle to the gusset plate, allowing more rivets to be used, is also seen to be incorrect, since practically none of the stress is transmitted into the angle through the rivets in the lock angle, at any rate until the member is near to the breaking load, when it is of no importance. The tests of McKibben [12] have shown that the effect of lock angles upon the breaking load is uncertain and not great in any case.

It thus appears that lock angles are of very little practical value and are unnecessary, expensive and cumbersome additions to the end connections.

§5. (c) *The Effect of a Change in the Line of Loading of the Gusset Plate.*

In order to obtain some idea of the relative importance of this factor, the load axis on the end plate of Specimen II was altered by reboring the pin-hole and adding reinforcing plates, the change being from a line 0.85 inch from the corner of the angle,—*i.e.*, in line with the centroid of the section,—to one in line with the rivets, 1.75 inches from the corner. The effect of this upon the load axis in the specimen may be seen from the annexed table. There was practically no change at the central sec-

[12] *Proc. Am. Soc. Test. Mat.*, vol. 6, p. 267; vol. 8, p. 287.

tion, but at the end the arm of the fixing couple was decreased 0.10 inch perpendicular to the end plate and increased 0.21 inch parallel to the end plate. Thus only the tertiary stresses were affected by the change. A comparison of the load axes in Specimens I and II (Table XIV), which were practically similar (although one was a little heavier than the other), except that the load axis in Specimen I was in line with the back of the angle while in Specimen II it was in line with the centroid, as explained above, reveals a change of 0.08 inch in the arms of the restrain-

TABLE XIII.

Specimen II.—Effect of a Change in the Load Axis.

Load, pounds	x_k (inches)			y_k (inches)		
	Axis in line with centroid	Axis in line of rivets	Change	Axis in line with centroid	Axis in line of rivets	Change
Central Section.						
5,000...	1.00	1.00	0.00	0.57	0.57	0.00
10,000...	1.01	1.00	0.01	0.55	0.55	0.00
15,000...	1.00	0.99	0.01	0.54	0.54	0.00
20,000...	1.00	0.99	0.01	0.54	0.52	0.02
		Mean.....	0.01		Mean.....	0.00
End Section.						
5,000...	0.99	1.08	0.09	0.65	0.40	0.25
10,000...	0.96	1.08	0.12	0.61	0.41	0.20
15,000...	0.97	1.07	0.10	0.61	0.40	0.21
20,000...	0.96	1.06	0.10	0.58	0.41	0.17
		Mean.....	0.10		Mean.....	0.21

ing couples at the central section and a smaller change at the end. In this case the differences may, of course, have been partly due to other causes than the difference in position of the axis, but in both cases they were so small as to make it appear that the exact position of the load axis on the end plate is not of very much importance, the distribution of stress being practically fixed by the line of rivets and the stiffness of the gusset plate. The effect on the tertiary stresses may be greater, but does not seem worthy of further investigation in view of its smallness in comparison with that due to the stiffness of the end plate in its own plane.

TABLE XIV.

Comparison of the Load Axes in Specimens I and II.

Load, pounds	x_1 (inches)			y_1 (inches)		
	Specimen I	Specimen II	Difference	Specimen I	Specimen II	Difference
Central Section.						
5,000...	0.92	1.00	0.08	0.49	0.57	0.08
10,000...	0.90	1.01	0.11	0.48	0.55	0.07
15,000...	0.90	1.00	0.10	0.46	0.54	0.08
20,000...	0.92	1.00	0.08	0.47	0.54	0.07
		Mean.....	0.09		Mean.....	0.08
End Section.						
5,000...	1.00	0.99	0.01	0.60	0.65	0.05
10,000...	1.02	0.96	0.06	0.61	0.61	0.00
15,000...	1.03	0.97	0.06	0.62	0.61	−0.01
20,000...	1.04	0.96	0.08	0.61	0.58	−0.03
		Mean.....	0.05		Mean.....	0.00

§6. *Further Remarks on the Double Angle.*

It is the usual practice in designing a member, such as Specimen III, made up of two angles back to back, to consider that bending occurs about a neutral axis parallel to the unconnected legs, the whole member bending as one piece. It has already been shown by the writer [13] that this is incorrect and that the only true way in which to regard this and other similar built-up sections is to consider that each element of the section tends to bend about an individual neutral axis, being restrained more or less by the end or other connections. It is only necessary to remark here that the present experiments bear out this theory and also show that a stiff end plate produces such large constraining moments that almost complete fixing results, the greatest deviation of the load axes of the two angles from their centroids in Specimen III being about 0.1 inch. How far this fixing may be relied upon in any particular case it is difficult to say, and the subject is worthy of further investigation.

§7. *General Conclusions.*

The above analysis leads to the conclusion that, with a gusset plate connection of the usual type, wide and rigidly connected,

[13] *Loc cit.*

the chief factor influencing the distribution of stress over the cross-section of the member is the stiffness of the end plate in its own plane. In comparison with this the effect of lock angles is practically negligible, and a considerable change in the line of pull on the gusset plate may be made without appreciably altering the stress distribution.

The bearing of the above upon the design of single- and double-angle members is obvious, although it is difficult to formulate exact rules. It may, however, be definitely stated that lock angles are of little, if any, value, and this is perhaps the most important result of the investigation. As to the correct ratio of maximum to mean stress to be used in designing, the earlier experiments of the writer [14] have shown that with long, narrow gusset plates, unconnected at the sides, there is practically no end restraint, no matter what the thickness, within practical limits, of the plate may be, and that a fairly good approximation to the actual distribution of stress may be obtained by considering the load axis as coinciding with the line of rivets and lying slightly within the end plate. A broad, stiff connection, however, is generally advisable, and with this the ratio of maximum to mean stress is much, say 30 per cent., lower than would be given by the above rule. It is difficult to estimate exactly, however, and perhaps the same rule might be used taking a higher value for the working stress, since the tertiary stresses would also be covered by it. With a narrow plate, however, the ordinary working stress should be used, since the tertiary stresses are not included. There remains, of course, the possibility of exceeding the yield point at certain fibres without danger. This, however, it would not be safe to rely upon in the present state of experimental knowledge of the subject.

The experiments on double-angle members show that such a member with a stiff end plate is an excellent type in practice, the stress being almost uniformly distributed at the central section, but, unfortunately, it seems to be impossible to predict exactly what the distribution will be in any particular case.

[14] In the earlier paper (*loc. cit.*) it was stated that the end plate had little constraining effect. This should be modified, in the light of the present experiments, to the statement given above.

§8. *Summary and Conclusion.*

The chief conclusions to which the present paper leads are:

1. That the only practicable experimental method at present available for investigating the distribution of stress in built-up members is by means of some form of extensometer, and that the simplified mirror extensometer used in the tests described is very suitable for this purpose.

2. That the assumption of a planar distribution of stress is justifiable in such members as are considered here, except perhaps close to the end connections, and that the ordinary theory may therefore be applied to an analysis of the distribution of stress in these members.

3. That in single- and double-angle tension members connected at their ends by means of rivets to wide and rigidly held gusset plates the stiffness of the gusset plate in its own plane has a considerable effect on the distribution of stress in the member, there being in every case a particular stiffness which will give the least maximum stress in the member for a given load.

4. That in such members lock angles are of very little, if any, value for the purpose either of giving a more equable distribution of stress in the member or of increasing the effective length of end connections.

5. That a slight change in the line of application of the load to the gusset plates does not materially affect the distribution of stress in the member, except possibly close to the end connections.

6. That the experiments on double angles bear out the theory that such members do not act as a single piece bending as a beam.

7. That the experiments lead to certain rules for design as formulated in Part III, §7.

In conclusion the writer wishes to thank Professors H. M. Mackay and E. Brown, of McGill University, for their personal interest and advice; and Mr. S. D. Macnab, of the McGill Testing Laboratory, who was associated with him throughout in the experimental work.

McGILL UNIVERSITY, Montreal,
January, 1915.

MODERN THEORIES OF MAGNETISM.*

BY

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As I must frequently employ certain words which are not in familiar use, it seems desirable that I should at first endeavor to make their meaning clear. Permit me, therefore, to recall to your minds some of the phenomena of magnetism. I take this steel bar and dip its end into this pile of nails. The clinging of the nails to the intruder shows us that it is magnetized. I bring an end of the bar near the north end of a compass needle and you notice a distinct manifestation of force, now attraction and again repulsion when I cause the other end of the bar to approach the same end of the needle. Both ends of the bar magnet are alike in picking up the nails, but they differ in their effect upon the compass needle. We thus see that there are two kinds of magnetism. I give the bar an opportunity to follow its own bent. You perceive that it faithfully turns into an approximately north and south line. Marking with chalk the north-seeking end of the bar, I bring it first to one end, then to the other end of the compass needle. I reverse the bar and apply its south-seeking end. You have seen both attraction and repulsion, and if you have noticed carefully you may have drawn the conclusion that north and south poles attract, while like poles (north and north or south and south poles) mutually repel.

A rough experiment shows us that the forces between two magnetic poles, as are called the parts of the magnet which attract and repel most strongly, diminish rapidly as the distance between them is increased. Keeping this fact in mind, let us put the bar magnet somewhere near the north pole of the needle and proceed to consider why the latter is deflected. It is attracted by the south pole of the bar. It is repelled by the north pole of the bar. The two poles of the bar, if it is an orthodox magnet, are equally strong. Hence we must conclude that the

* Presented at the joint meeting of the Electrical Section and the Philadelphia Section, American Institute of Electrical Engineers, held April 1, 1915.

pole of the bar magnet nearer to the needle has the preponderating influence just because it is nearer, and not because it is the stronger. It is opposed by its opposite pole, which is striving to do just the opposite thing to the needle, but which fails to win because it is too far off. If it were possible to keep the nearer pole just where it is and then to pull the magnet out along its length, as a candy-maker does taffy, until it were twice as long as at first, though both poles remain unchanged, then the nearer pole would be opposed more feebly by the distant pole and the needle would be deflected through a greater angle. Or, on the other hand, could the bar magnet dwindle in length until its two ends were only an inch apart, though no change in the strength of the poles had occurred, then the two antagonistic poles would be at so nearly the same distance from the end of the needle that their opposing attraction and repulsion would be almost equal, and their resultant would be small.

From these considerations it becomes evident that something more than the strength of the poles of a magnet must be known in order to determine its effects upon other magnets. The additional factor is the distance between its poles. The greater this distance the greater the resultant force exerted. The product of pole-strength by distance between the poles is designated the moment of the magnet. Just as in a lever you can double the equilibrating effect of a weight by removing it to twice the distance from the fulcrum, so in the case of a magnet you can double its force upon distant magnets by doubling the distance between its poles.

We could thus have two magnets of equal moment, one a metre from pole to pole, the other only half a metre. The first would have poles only half as strong as the second. So far as attracting or repelling a distant magnetic needle is concerned, these two magnets would be equivalent, yet the longer would have twice the volume of steel in it, if their cross-sectional areas were the same. In the smaller magnet the same moment is obtained with only half the volume of steel. In other words, if we divide the moment of each magnet by its volume we shall get quotients which are different and are as 1 to 2. This quotient is called the intensity of magnetization and designates how much moment the magnet has for each unit of volume. Should we divide the moment by the mass of the magnet the quotient would tell

how much moment there is for each unit of mass, for each gramme. In this case the quotient is designated the specific intensity of magnetization.

How was the bar of steel magnetized? Perhaps by putting it inside a coil of wire in which an electric current was flowing. Before the bar was inserted there was a magnetic field within the coil. Let its strength be called H units; *i.e.*, let there be such a condition there that a unit magnetic pole would experience a force of H dynes. When the steel bar is inserted in this field it becomes magnetized. It has two poles, each of a certain strength. It has therefore acquired a magnetic moment, and has a corresponding intensity of magnetization and also a specific intensity of magnetization. These effects are due to the field strength of H units. The quotient got by dividing the intensity of magnetization by H is called the susceptibility of the specimen of steel. The corresponding quotient got by dividing the specific intensity of magnetization by H is called the specific susceptibility, and is designated by the Greek letter χ.

I have now finished the definition of technical terms.

There are good reasons for believing that the molecules of steel are all individual magnets, and are so all the time, before the bar has been magnetized as well as afterwards. The process of magnetization is thought to consist in bringing about an orderly arrangement in the mass of tiny magnets which had been located in an entirely haphazard way, so that as many north poles had pointed in any given direction as south poles, with the result that the opposite poles neutralized each other in effect. When the rod of iron or steel is put in a magnetic field,—that is, near a permanent magnet or in a coil carrying a current,—the magnetizing force causes the little molecular magnets, in part at least, to turn so that their axes lie in the direction of the magnetic lines of force. When the rod is taken away from the field of force it still keeps some magnetism, but if it is struck or heated so that the molecules are stirred up they resume their fortuitous arrangement and the rod loses a part of its magnetism.

There are three classes of substances in respect to magnetic behavior.

1. *Diamagnetic.*—Such as are repelled by a magnet and seek to set themselves with their axes at right angles to the lines of force or, more accurately, seek to move from the strongest to the

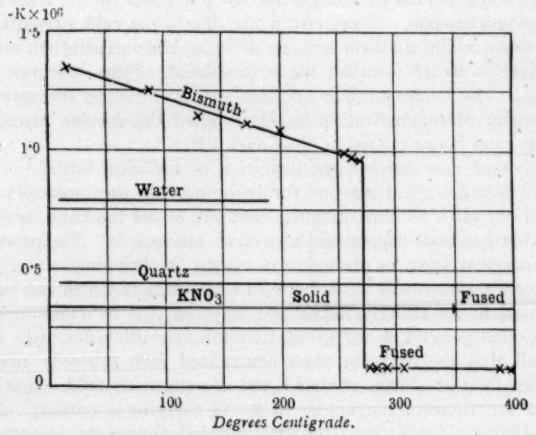
weakest part of the magnetic field. The force called forth, however, is always small. Bismuth exhibits diamagnetism to the highest degree.

2. *Paramagnetic.*—Such as are feebly attracted by a magnet and strive feebly to move from the weakest to the strongest part of a magnetic field. Oxygen and platinum are examples.

3. *Ferromagnetic.*—These are iron, steel, nickel, cobalt, and some alloys. They are strongly attracted by a magnet.

Diamagnetic substances are most numerous, then come paramagnetic, and, finally, the small but important group of ferro-

FIG. 1.



magnetic substances. This last class is best known because its members exert a much greater magnetic force than do representatives of the two other groups.

As the history of magnetism runs back to the time when the mythical Greek shepherd felt the nails of his shoes cling to the rock, I must set an arbitrary limit to the extension of the word "Modern" in the title of my lecture. I will therefore begin twenty years ago. In 1895 Pierre Curie, whose work in connection with radium has eclipsed his other excellent physical investigations, published an important paper on magnetism. He

set himself the problem of studying the magnetic properties of diamagnetic, paramagnetic, and ferromagnetic substances when the temperature and the intensity of the magnetic field changed. His temperature range was from room temperature to 1370° C. Let us look at some of his results with diamagnetic substances. The ordinates represent specific susceptibility, while the abscissæ are temperature. Notice that water, quartz, and saltpetre keep the same susceptibility over a wide range of temperature. The water was in a closed vessel and was thus kept in a liquid state up to 199° C. The comportment of bismuth is worthy of your consideration. At 20° it has a susceptibility of -1.3×10^{-6}. As the temperature rises this falls linearly until at 273° it has dwindled to $-.96 \times 10^{-6}$. At this temperature the bismuth melts and the susceptibility of the liquid is only one-twenty-fifth of that of the solid at the same temperature. When the temperature of the molten substance is still further raised it comports itself just as do the other substances shown on the slide; *i.e.*, it shows a susceptibility independent of temperature. Saltpetre shows no change of susceptibility upon melting.

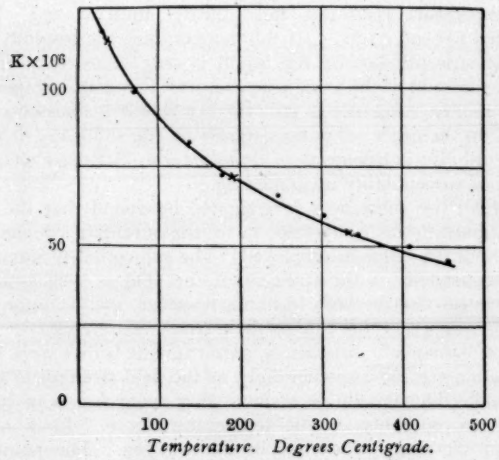
For all the substances investigated he found that the magnetic moment was proportional to the strength of the field throughout the range he employed. The susceptibility was therefore independent of the strength of the field as well as of the temperature. Let us keep in mind, however, that antimony and bismuth transgress this rule as their temperature rises.

Like diamagnetic substances, paramagnetic bodies were found to maintain the same susceptibility as the field strength changed, but, unlike diamagnetic substances, they were found to have a smaller susceptibility as the temperature rose. Look at the diagram showing Curie's results for oxygen. The points indicating the experimental results do not lie on a straight line. Let the temperatures be changed from the Centigrade to the absolute scale by adding 273° to the former reading. Then take the susceptibility at any temperature and multiply it by that temperature on the absolute scale. The product will be found to be about 33700×10^{-6}. For example, at 75° C. = 348° abs. the susceptibility is 98×10^{-6}. The product $348 \times 98 \times 10^{-6} = 34104 \times 10^{-6}$. For 452° the corresponding product is $(452 + 273) \times 46.9 \times 10^{-6} = 34002.5 \times 10^{-6}$. This agreement of the two products is striking, especially as the difference in temperature

of the two observations was 377° C. The name "Curie constant" has been given to this constant product.

Curie phrases the law thus: "From 20° to 450° C. the specific susceptibility of oxygen varies inversely as the absolute temperature." The same law holds approximately for the metal palladium and for several salts in solution. As has been said, it was found in the case of diamagnetic substances that a change in the strength of the magnetic field made no difference in the specific susceptibility. We note then that this property of dia-

FIG. 2.



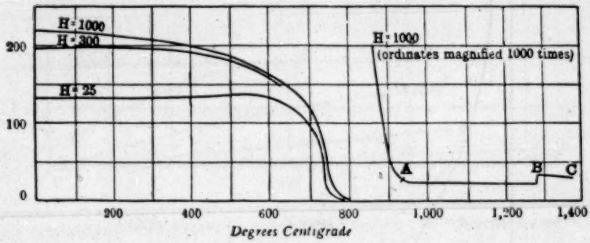
magnetic and paramagnetic substances is for both independent of the field strength, but does depend upon the temperature for paramagnetic substances, while it does not so depend for diamagnetic bodies. This difference in behavior led Curie to conclude that "these results are in favor of theories which attribute paramagnetism and diamagnetism to causes of different natures."

It has been found, however, that even for oxygen the law connecting temperature with susceptibility changes as low temperatures are reached. Moreover, Du Bois and Honda found

paramagnetic substances in which a change in temperature made no change in the susceptibility.

Curie extended his experiments to the third class of substances, viz., ferromagnetic, and found many interesting things. This curve traces the magnetic history of soft iron from 0° C. to beyond 1300° C. To the left are three curves for different field strengths. Notice how nearly parallel to the horizontal axis the $H = 25$ curve runs. Since the ordinates here give intensity of magnetization, while the abscissæ give temperature Centigrade, this parallelism means that with this strength of field the piece of iron maintained the same intensity during a rise of temperature of several hundred degrees. However, when the strength of the field was 1000 units the intensity fell from the first as the tem-

FIG. 3.

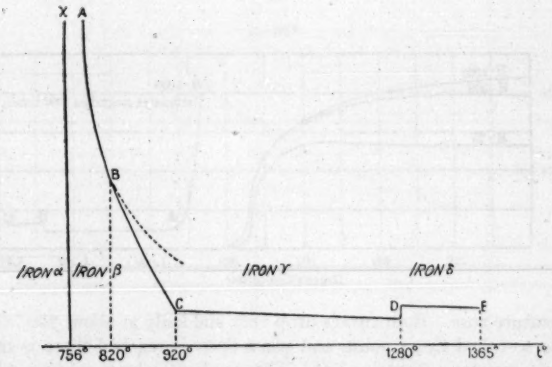


perature rose. Both curves drop very suddenly at about 750° C., the so-called Curie point, and when 800° is reached there is but little magnetic intensity left. This explains why a magnet will not pick up a white-hot nail, but will pick it up when it has cooled to perhaps 700°. The right-hand curve has its ordinates magnified 1000 times. From 800° to about 860° they are too great to be represented on the diagram. At *A*, 930°, the slope grows less and is very slight up to *B*, 1280°, where all at once the intensity becomes fifty per cent. greater. After this the curve again slopes downward. In the left of the diagram iron is ferromagnetic. At about 750° it changes continuously into a paramagnetic substance whose course is traced in the right of the figure. This change from ferro- to paramagnetic occurs for the mineral magnetite at 530° and for nickel at 340°.

The following figure shows the same facts in a different way. The ordinates are specific susceptibility. Alpha iron, beta iron, gamma iron, and delta iron are limited by the temperatures 756°, 920°, 1280° C. When iron ceases to be strongly magnetic at 756° and its temperature is further raised, the susceptibility falls rapidly, as the curve shows. In the gamma state the decrease of susceptibility is very gradual. At 1280° C. there is a sudden rise, after which again a gentle decline occurs.

So far we have considered the connection between magnetization intensity and temperature. Let us give attention to the curve showing change of intensity with field strength. The

FIG. 4.



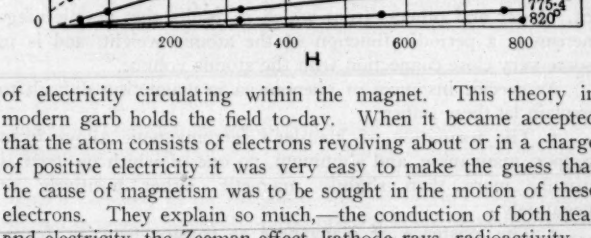
ordinates here represent the former quantity and the abscissæ the latter. When the temperature of the specimen of iron is even as near the Curie point as 740.5° there is considerable rise in intensity of magnetization as the field strength increases. An elevation of temperature of a few degrees brings down this increase to a very modest amount. Compare the intensities for $H = 320$. An increase of 3.7° from 740.5° to 744.2° C. reduces the intensity to about half its value at the lower temperature, while the intensity for 820° is so small that it cannot be represented on the diagram. Notice further that the curve for 740.5° becomes a straight line for 749.9° and higher temperatures.

Curie pointed out the similarity in the form of the curves connecting, on the one hand, the density and temperature of a gas near the critical state and, on the other, the intensity of magnetism and temperature of iron near the Curie point. This analogy has been of use both in suggesting methods of expressing results and also in pointing to explanations of phenomena.

So far you must agree with me that I have failed to justify the title of this lecture as "Modern Theories of Magnetism." No theories, but only facts, obtained by experiment, have been presented.

In 1820, less than a month after Arago, before an audience of the French Academy including Laplace, Biot, and Ampère, had demonstrated Oersted's experiment proving an electric current to produce magnetic effects, one of that audience, Ampère, made the suggestion that magnetism might be due to currents

FIG. 5.



of electricity circulating within the magnet. This theory in modern garb holds the field to-day. When it became accepted that the atom consists of electrons revolving about or in a charge of positive electricity it was very easy to make the guess that the cause of magnetism was to be sought in the motion of these electrons. They explain so much,—the conduction of both heat and electricity, the Zeeman effect, kathode rays, radioactivity,—that they tempted the theorist to impose one additional burden upon them,—that is, the phenomena of magnetism. The first attempts were, however, not successful. Both Sir J. J. Thomson, in England, and W. Voigt, in Germany, discussed at length the motion of these negatively-charged electrons without finding an explanation for the facts of magnetism. It remained for a Frenchman, Langevin, to find the clue. In 1905, ten years after Curie's work appeared, he made public his solution of the problem of diamagnetic and paramagnetic substances.

It may perhaps be well for us to recognize how complicated the problem of magnetism is in order that too much may not be expected at first of any explanation. I quote here from G. A. Schott [1]:

"1. Effects due to the mutual interactions of the molecules of the substance; these are usually taken to include hysteresis and coercive force, and the numerous interactions between strain and magnetism.

"2. Effects due to the actions between the atoms of a molecule. The smallness of the magnetism of oxides and salts of iron and cobalt in proportion to the amounts of iron or cobalt contained in them is an effect of this kind.

"3. Effects due to the constitution of the atom itself. The fact that the amalgams of iron and cobalt are as strongly magnetic in proportion to their concentration as the pure metals, coupled with the usual supposition, made on chemical grounds, that the molecule of a metal is monatomic, can hardly be explained except by supposing that magnetism is, in the last resort, an atomic property, and that the largeness of the magnetism of iron and its congeners is due to some peculiarity of atomic structure. This conclusion is supported by the experiments of St. Meyer and others, from which it results that atomic magnetism is a periodic function of the atomic weight, and is in some very close connection with the atomic volume."

As specific instances of phenomena in magnetism difficult to explain let these suffice:

1. The formation of Heusler's ferromagnetic alloys from copper, manganese, and aluminum, no one of which by itself is usually more than slightly magnetic, either positively or negatively.

2. The existence of manganese as paramagnetic or ferromagnetic according to past treatment.

3. The change of tin from a diamagnetic to a paramagnetic state. When gray tin, which is diamagnetic, is heated its negative specific susceptibility grows greater and at 35° C. reaches zero. It continues to rise with the temperature up to 50°. A further increase of temperature fails to affect it until the tin melts at 232°, when it suddenly drops to a value below zero and

[1] *Phil. Mag.*, Ser. 6, vol. xv, p. 172 (1908).

the liquid metal, a diamagnetic substance, retains this negative susceptibility as it rises still further in temperature.

4. The formation of iodide of mercury and potassium, a paramagnetic substance from three diamagnetic elements.

Surely no theory could hope to do more than explain the salient and general features of so complex a group of phenomena as magnetism presents to us.

LANGEVIN'S THEORY.

Every atom, according to current views of the constitution of matter, has a number of electrons revolving in closed orbits within it or about its centre. All electrons have equal charges of negative electricity. Our fellow-countryman, Henry A. Rowland, proved that a charge of electricity in motion produces magnetic effects just as a current does. Accordingly each electron moving in its orbit is equivalent to a current of electricity flowing in the same orbit. One face of the orbit is a magnetic south pole, the other a north pole, and magnetic lines of force surround the moving electron.

Let us fix our mental gaze on one such electron revolving in a circular path. Let a magnetic field be set up perpendicular to the plane of the orbit,—that is, so that a magnetic pole would be urged by the field to move at right angles to the plane of the orbit. There will be an interaction between the field of force and the electron in motion, with the result that the electron continues to revolve at the same distance from the centre of its path, but makes a revolution in a different time. As any change in the velocity of the electron will cause a change in the electric current to which it is equivalent, and as a change in the current will carry with it a change in the magnetic effect, it follows that a change in the magnetic moment of the orbit follows upon the arrival of the external magnetic field which affects the electron. Calculation shows that no matter in which direction the electron is revolving, the change in magnetic moment of the orbit is always in the same direction. For example, let the external field be that existing between the north and the south pole of a horseshoe magnet. The effect of this field upon an electron revolving in an orbit whose plane is at right angles to the line joining the poles of the magnet is to cause the orbit to strive to develop a north pole on its side toward the north pole of the magnet and a south

pole on the other side. This kind of induction effect is characteristic of diamagnetic substances. Even if the electron is at rest when the magnetic field is applied, the result is about the same, for Lorentz has shown that the electron will be made to rotate about an axis parallel to the direction of the field and in such a direction as to make its lines of force oppose those of the external field.

Perhaps enough has been said to show how closely the modern theory is in fundamental agreement with the ideas of Ampère and of Weber. In 1896 Zeeman, in Holland, discovered that a yellow sodium flame put in a strong magnetic field suffers a change in the character of the light which it emits. As the motion of the electrons is regarded as the source of radiation, it will be noted that the Zeeman effect is allied with the diamagnetic effect of a magnetic field outlined above. Since the Zeeman effect is found to occur in nearly all spectral lines of nearly all substances which have been examined, we are led to think that the diamagnetic effect of a magnetic field is very general; that it occurs in substances classed as paramagnetic or ferromagnetic as well as in diamagnetic bodies. Let us see how Langevin accounts for this.

Each atom has several electrons revolving in orbits. These orbits may be in distinct planes or may differ in other ways. The total magnetic moment of the atom will be found by summing up, with proper regard to direction and magnitude, the magnetic moments of the separate orbits. The result of this integration must give one of two results, either zero or some significant quantity. Langevin investigated the effect of the external magnetic field on an atom having no initial resultant magnetic moment. As the magnetic effects of the various electrons neutralized each other the atom as a whole would not rotate when an external field was applied, as it would tend to rotate were there a single orbit, in an endeavor to set the axis of the orbit in coincidence with the direction of the magnetic field. Even if the orbits were not perpendicular to the direction of the field the general effect would be unaltered. Although the total magnetic moment of the atom was zero before the imposition of the magnetic field, this does not cause to follow that the total change in the magnetic moment would be zero. On the contrary, the changes in the various electronic circuits are in the same

sense. Langevin's theory expresses the ratio of the change in the magnetic moment to the original magnetic moment of an orbit in terms of quantities whose magnitude is known. By using this relation it is found that the greatest attainable strength of magnetic field would change the magnetic moment of a diamagnetic atom by about one part in ten thousand.

Does Langevin's theory have anything to say about Curie's statement that the susceptibility of diamagnetic substances changes in general neither with temperature nor with field strength? With regard to the connection between susceptibility and field strength, his theory shows that the magnetic intensity is proportional to the strength of the field. Since susceptibility is the quotient of the first of these quantities by the second it follows that this quotient is constant. In relation to the connection between temperature and susceptibility let it be noticed that Langevin finds nothing to occur outside of the atom when the magnetic field is applied. The atom as a whole does not rotate nor receive motion of translation. The application of a magnetic field will not cause it to affect in a different way the motion of its neighbors, nor will it make them affect its motion differently. If both it and its neighbors rise in temperature their motion will be changed, but that increase of kinetic energy will not affect what goes on within the atom. In short, in a diamagnetic substance the application of an external magnetic field will not produce any thermal effect, nor will a change of temperature bring in its train any magnetic effect. The susceptibility will be independent of temperature, as indeed Curie showed it to be in several substances. As has already been pointed out, he found bismuth to be an exception to this rule, and more recently other investigators have increased the list of exceptions. Honda and Owen have found diamagnetic substances whose susceptibility increases with temperature, others in which that property diminishes with temperature in addition to those for which it remains constant during temperature change. Were bismuth the only exception, an explanation might be found in the effect of the magnetic field upon its free electrons, those electrons which give it electric and thermal conductivity. It is known that the free path of these electrons is much longer than is commonly the case.

As has been stated before, the motion of the electrons within the atom produces light. The fact that the wave-length of the emitted light is scarcely affected by a change of temperature is quite in line with the absence of an effect of temperature upon magnetic moment, since a change in the motion of the electron would affect both its light emission and its magnetic moment.

Let us turn next to Langevin's theory of paramagnetic substances. Here again the atoms have revolving electrons, each having a definite magnetic moment. In diamagnetic substances the summation of the moments of the various orbits gave zero. In both paramagnetic and ferromagnetic bodies Langevin assumes the summation to give a resultant moment which is not zero. What will occur when an external magnetic field is imposed on the atom? First, in every orbit there will be produced a change of magnetic moment just as if the body were diamagnetic. This effect will, however, be masked by what occurs after a very brief time. Just as a dipping needle will set its axis of magnetic moment along the direction of the earth's field, so will the atom strive to set the axis of its resultant magnetic moment along the direction of the applied magnetic field, and the atom will turn. If there were nothing to hinder, the atoms would line up with fine regularity, all the magnetic axes being parallel to each other and to the magnetic field, but this is not allowed to occur. The thermal agitation of the particles of matter will tend to undo any nice arrangement caused by the magnetic field. It will not succeed entirely, nor will the magnetic field. A compromise will be effected. A state of statistical equilibrium will be reached in which the same number of molecules per cubic centimetre will always have their axes in a specified direction, though this fixed number will not be made up of the same molecules at two different times. If the substance is a ferromagnetic solid, another cause will intervene to prevent the alignment of the atomic axes,—to-wit, the mutual effect of one atom on the other. This is of great importance, but its consideration must be deferred. It may be well to sum up what seem to be the distinguishing marks of the three magnetic states of matter. Diamagnetic substances consist of atoms having no resultant magnetic moment. Paramagnetic and ferromagnetic substances have atoms possessing a resultant moment. In ferro-

magnetic substances interaction between atom and atom or molecule and molecule plays a predominating part.

Let us revert to paramagnetic substances. When the external field is brought to bear upon them, first each atom experiences a change of moment producing a diamagnetic effect. This is followed by a turning of the atom so as to put its magnetic axis more closely in line with the lines of force of the magnetic field. Let us suppose that there is no thermal motion nor interatomic force to interfere; that is, let the experiment be made with a perfect gas at absolute zero of temperature. All the atoms would be in line and the substance would have the greatest magnetic moment per gramme possible for it to have. If heat were applied so that the temperature should rise, some of the atoms would get out of alignment and the magnetic moment and consequently the susceptibility would grow less. The application of a stronger field would tend to bring back the atoms to line and thus increase the moment. These elementary considerations show that as temperature goes up susceptibility goes down, and that magnetic moment and field strength increase together. These results are at least consistent with Curie's statement that the susceptibility of some paramagnetic substances is independent of the field strength and inversely proportional to the absolute temperature.

Langevin, however, goes much further than such a general agreement. He deduces an equation for a paramagnetic gas connecting intensity of magnetization, temperature, and strength of field. For field strengths from zero to those enormously greater than any we can now produce this equation shows the intensity proportional to field strength, that is, the susceptibility to be constant while the temperature is constant, and also the susceptibility to vary inversely as the temperature. Note that this derivation holds for a gas or for a dilute solution and not for solids and liquids in general. Before Langevin, DuBois deduced Curie's law that susceptibility is inversely as the absolute temperature from theoretical considerations. There can be no doubt that Langevin advanced magnetic theory greatly, though he left much unexplained.

Ferromagnetic substances differ from paramagnetic ones in these respects: they are capable of being magnetized so as to

have a very much greater magnetic intensity than the latter ever have; they form permanent magnets, and their present magnetic state depends on their past magnetic history. Their comportment with regard to magnetic forces is characterized by an almost total lack of the simplicity which marks both diamagnetic and paramagnetic bodies. It remained for Pierre Weiss, now professor in the Federal Polytechnikum in Zürich, to unravel in part the mysteries of their behavior. Professor Weiss has, I believe, done more than any other living man to make clear the physiology and anatomy of the magnet. In 1907 a committee of the French Academy of Science, consisting, among others, of Mascart, Lippmann, Becquerel, Amagat, Violle, and Poincaré,

FIG. 6.

PIERRE WEISS.

awarded him the Kaštner-Boursault prize, and in the present year he has become the recipient of the Lasferre prize of $1600.

Weiss's method of attacking the problem of ferromagnetic substances is to extend to them the theory which Langevin had framed for paramagnetic bodies. To do this he supposes that there is within a ferromagnetic substance everywhere and at all times, when it is magnetized as a whole, a magnetic field due to the influence of the adjacent elementary magnets. This field does not manifest itself outside of the substance because there the distance from the centres of force is too great, and this force, unlike gravity and other forces, decreases not inversely as the

square but as the sixth power of the distance. This last relation Weiss deduces on the supposition that the action proceeds from one point in a molecule and not from two points or poles in the same molecule.

He assumes the molecular field, as he calls it, to have a strength proportional to the intensity of magnetization of the material and to be in the same direction. When a piece of iron is put in a magnetic field there is within it a total field whose strength is the sum of the imposed field and of the molecular field.

The process of magnetizing a piece of steel would probably be about as follows: To begin with, each atom is a magnet because of the motion of its electrons. Each atom experiences a magnetic field coming from its neighbors, but from atom to atom these molecular fields change in direction, being distributed according to the laws of chance. Let the steel be placed in the middle of a coil of wire traversed by a current. The imposed magnetic field causes the atoms of steel to turn so that their magnetic axes are as nearly in line with the direction of the external magnetic field as their thermal agitation will permit. When the atoms turn into line the "molecular field" emanating from them does the same. Let the current in the magnetizing coil cease. The molecular field within the steel will retain a considerable part of its former value. The atoms will thus remain in at least partial alignment under the action of their mutual forces. This explanation of permanent magnetism is close to that given by Ewing.

Though as yet there seems no way of directly measuring the strength of this molecular field, it can be calculated and enormous magnitudes are obtained, no less than 17,300,000 gausses for magnetite and 3,030,000 for Fe^2Ni respectively. Weiss has examined the possibility of such enormously strong fields arising from rotating electrons in atoms. He concludes that it is impossible. It may be this which has led him to revise his conception of the molecular field. In 1914 he writes: "It is convenient to represent the mutual actions between molecules which come in play in magnetic phenomena as being themselves magnetic. This is, however, by no means necessary. Whatever may be their nature, the effects would be the same,"—under a certain proviso. He proceeds to argue that the actions are neither

magnetic nor electrostatic in character, and frankly acknowledges that the nature of the force is as yet unknown. Even in spite of this limitation in our knowledge of it, Weiss says: "The hypothesis of the molecular field has shown itself fertile. At once this field, joined to the external magnetic field, permits us to derive the strong magnetization of ferromagnetic substances from the laws of paramagnetism alone. This is not without analogy with the properties of fluids, translated by the theory of Van der Waals, where the great density of liquids is derived from the properties of gases by means of the interior pressure which comes to join itself to the pressure from outside. Just as a liquid can exist under zero external pressure, and even under a negative pressure,—that is to say, under the internal pressure alone,—so can a ferromagnetic body be strongly magnetized by the molecular field alone in the absence of any external field. . . . The molecular field has given for the first time a theoretical deduction of the law of the thermal variation of magnetization at saturation. . . . The conception of the molecular field leads to an easily verifiable law for the magnetization of ferromagnetic substances above the Curie point." This last statement is of special interest. A hypothesis is surely valuable when it leads to the discovery of a hitherto unsuspected law. This is the law: the specific susceptibility of a ferromagnetic substance at any temperature above the Curie point, multiplied by the excess of that temperature above the Curie point, gives a constant. For a considerable number of ferromagnetic substances studied, including both metals and alloys, this relation has been found to hold. The mineral pyrrhotine and gamma iron are exceptions.

The constant product obtained as above indicated is found to be related to the moment of a single molecule in just the same manner as the Curie constant is related. Hence it also may be called by the same name.

By using the theory of the molecular field Weiss has been able to explain why it is that the specific heat of ferromagnetic substances suffers an abrupt change in amount at their Curie points. In a permanent magnet the atoms are in such positions that their potential energy due to mutual attractions and repulsions is a minimum. If an atom be rotated slightly it will be necessary to do work against the restraining forces. This is confirmed by

the fact that the impartation of heat energy to a magnet causes its magnetic intensity to grow less, due, we believe, to a change in the alignment of its molecules. The specific heat of the substance may be regarded as made up of two parts, one characteristic of the substance and the other due to the molecular field within it. The enormous value of the molecular magnetic field makes this second term in the sum of the same order of magnitude as the specific heat itself. It can be shown that the energy which a magnetized body has is proportional to the square of its intensity. The specific heat due to magnetization will then be proportional to the change in the square of the intensity caused by a change of 1° in temperature. As the temperature rises and approaches the Curie point this change in the square of the intensity becomes greater and greater, and the specific heat due to magnetism grows correspondingly greater. Since the molecular field disappears when the Curie point is reached, there will be a marked difference between the specific heat above that point and below it. The following table [2] contains some data for iron calculated from Curie's experimental results. The first column gives the temperature of the iron, the second its specific intensity, the third, the number of calories absorbed for magnetic effects per gramme in raising the temperature from the stated temperature to the Curie point, the fourth, the specific heat in calories due to magnetic effects, and the last column the temperature interval for which the specific heat was measured.

t° C.	$\frac{I}{D}$ C.G.S.	Cal.	Cal.	In the interval.
20	216.3	16.8	.005	20°–275°
275	207.5	15.5	.013	275°–477°
477	189.6	12.9	.027	477°–601°
601	164.0	9.7	.045	601°–688°
688	127.0	5.8	.068	688°–720°
720	100.7	3.6	.108	720°–740°
740	64.	1.5	.136	740°–744.6°
744.6	50.1	.9		
753.	0 (by extrapolation)			

It is possible to locate the Curie point both by magnetic and by thermal experiments, and to determine the difference between the specific heats just above and below the Curie point in the

[2] E. H. Williams, "The Electron Theory of Magnetism," p. 54.

same two ways. The following table shows an excellent agreement between the results of the two methods of determination:

	By magnetic measurements.	By heat measurements.
Iron.		
Discontinuity	.136 cal.	.112 cal.
Curie point	753° C.	758° C.
Nickel.		
Discontinuity	.025 cal.	.027 cal.
Curie point	376° C.	376° C.
Magnetite.		
Discontinuity	.048 cal.	.050 cal.
Curie point	588° C.	580° C.

THE MAGNETON.

Following the ideas of Ampère and Weber, we have considered the individual atom as having magnetic poles and, in consequence, a magnetic moment. This "molecular moment" (as it will here be called, in accordance with the practice of P. Weiss, though "atomic moment" might be a more logical designation) is a quantity of considerable interest. How can it be measured?

Let us suppose ourselves endowed with eyes and fingers adequate for the task of handling single atoms. Let us take a thousand of them and arrange them in a row, with all the north poles toward the same side. This would mean that the north pole of one atom would lie next to the south pole of the adjacent atom. Since these two poles are of equal strength and very close together, they will neutralize each other for points a little distance away. The north pole at one end of the row and the south pole at the other end, however, are not so neutralized. The magnetic moment of the row is equal to the strength of one of the uncompensated poles at the end multiplied by the length of the row,—that is, by 1000 times the length of one atom. The moment of a single atom is the strength of one of its poles multiplied by the distance from pole to pole, or the length of the atom. The magnetic moment of the row of 1000 atoms will be 1000 times as large as the moment of a single atom, because the pole strength is the same in calculating both quantities, while the distance from pole to pole is 1000 times as long in the row as in the single atom. We see then that if we can take a very large

number of atoms and line them up and keep them from getting out of line by reason of heat agitation we will be able to measure the magnetic moment of the resultant body of tangible size. The moment of the single atom can then be calculated by dividing the total moment by the number of atoms in the body.

An impressed magnetic field will bring about the alignment, and a reduction of temperature will do away with disturbance of the arrangement by heat. In more technical language, the phenomena of magnetic saturation at the absolute zero must be studied. In 1910 Kamerlingh Onnes and Weiss made public the result of their studies of several ferromagnetic substances almost under the conditions just specified. They gave the magnetic moments per gramme atom,—that is, for a quantity of the substance whose mass is one gramme multiplied by the atomic weight of the substance. A gramme-atom of an element has the same number of atoms in it, no matter what the element is. They found the magnetic moment of iron per gramme atom to be 12360 (C.G.S.), and of nickel 3370. If the first of these numbers be divided by 11 the quotient is 1123.6. The second divided by 3 gives practically the same quotient, 1123.3. If we compare a mass of iron saturated at very low temperature with a mass of nickel having in it as many atoms as there were in the iron it will be found then that the iron has a moment $\frac{1}{3}\frac{1}{3}$ as great as the nickel, and hence that the iron atom has a magnetic moment $\frac{1}{3}\frac{1}{3}$ as great. There are 68×10^{22} (*i.e.*, 68 followed by 22 ciphers) atoms in one gramme-atom. Divide the moment of a gramme-atom of iron by the number of atoms in it and the quotient is 181.7×10^{-22}. This is the magnetic moment of one iron atom. By a like division the same quantity for a nickel atom is found to be 49.5×10^{-22}. The greatest common divisor of these quantities, which goes 11 times into the first and 3 times into the second, is 16.5×10^{-22} (C.G.S.). To this quantity of magnetic moment Weiss, in 1911, gave the name "magneton." About six months previously L. A. Bauer, the director of the Department of Research in Terrestrial Magnetism of the Carnegie Institution, had used the word in a different sense.

The investigators were unable to saturate cobalt magnetically and thus obtain its magnetic moment per atom. By an indirect method,—that is, by studying the moment at saturation of nickel-

cobalt alloys of varying composition,—Bloch found the moment of the cobalt atom to be 8.925 magnetons, while Preuss got from the study of iron-cobalt alloys the value 8.97 magnetons. Another interesting result of Preuss's work is the discovery of an iron-cobalt compound, 34 per cent. Co + 66 per cent. Fe, Fe^2Co, which is more magnetic than iron to the extent that at ordinary temperatures its intensity of magnetization exceeds that of iron by 10 per cent. When Weiss used less than one kilogramme of this material to make pole pieces for his electromagnet he got a field of 55,170 gausses, whereas under like conditions with iron pole-pieces the field was only 52,580 gausses. When the electromagnet was energized by a power of 5.5 kilowatts with the new pole pieces the same field was obtained as with four times as many kilowatts and the iron pieces.

For the determination of the atomic moment at high temperatures there is fortunately a simple relation between this quantity and the Curie constant.

Nickel has been well studied. It has been found to exist in two states: β_1 nickel from 370° to 900° with eight magnetons per atom, and β_2 nickel above 900° with nine magnetons per atom. The following table sums up the work done on this element:

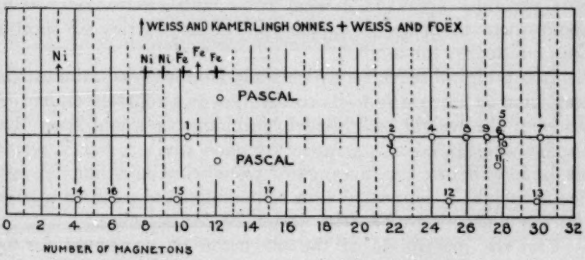
Experimenter	Substance	Number of magnetons per atom
Weiss and Foëx	Ni.β_1 directly	8.03
Bloch	Ni.β_1 directly	7.99
Renker	Ni.β_1 directly	8.04
Renker	Ni.β_1 study of ferronickels	8.05
Weiss and Foëx	Ni.β_2 study of ferronickels	9.03
Bloch	Ni.β_2 directly	8.96
Renker	Ni.β_2 directly	8.93
Renker	Ni.β_2 study of ferronickels	8.96
Bloch	Cobalt directly	15.01
Preuss	Cobalt directly	14.99
Weiss and Foëx	Iron β_1 directly	12.08
Renker	Iron β_1 directly	11.98
Renker	Iron β_1 study of ferrocobalts	11.97
Weiss and Foëx	Iron β_2 directly	10.04
Preuss	Iron β_2 directly	10.00
Renker	Iron β_2 directly	10.05
Renker	Iron β_2 study of ferrocobalts	10.10
Preuss	Iron β_2 study of ferrocobalts	20.05
Renker	Iron β_2 study of ferrocobalts	20.20

In this diagram the vertical lines are drawn one magneton apart. The circles and arrows indicate by their position horizontally the number of magnetons in the molecule of the substance. It is to be noted how often the circles come on the vertical lines and how rarely they miss them.

A study of magnetite shows that it exists in five states in which the molecular moments are in the relation 4 : 5 : 6 : 8 : 10, but seem not to be exact multiples of the magneton. Even in the case of solid paramagnetic salts the atomic moment keeps close to whole multiples of the magneton.

From considerations based on its specific heat it seems essential to conclude that the oxygen molecule is rigid and has only a

FIG. 7.



1–11. Various salts of iron.
12. Cobalt chloride.
13. Manganese sulphate.
14. Permanganate of potassium.
15. Cupric sulphate.
16. Cuprammonium sulphate.
17. Uranous sulphate.

single magnet. Its moment is found to be 14.014 magnetons. For the molecule of NO the moment is 9.039 magnetons.

The same atom seems capable of having very different magnetic moments according to its temperature and associates within the chemical molecule; *e.g.*, near the absolute zero a nickel atom has a moment of three magnetons, above 400° it has eight, above 900° it has nine, while in solutions of its salts it has sixteen.

Weiss says: "If we admit, what is extremely probable, that the magnetic moment resides in a material substratum, we can say that the magneton is a constituent element of a great number of magnetic atoms and probably of all. So far proof has been made for Fe, Ni, Co, Cr, Mn, V, Cu, U. . . . The magneton

idea furnishes a new point of attack on magnetic problems. The parallelism of intensity of chemical properties with the number of magnetons raises new questions. What rôle do the magnetic phenomena play in chemical combination? Are chemical forces in certain cases the attraction of elementary magnets? Does valence stand in any relation to the magneton? . . . The magneton is a third constituent of matter which, like the electron and the alpha particle, belongs to a great number of molecules. Antiquity believed in the unity of matter. The alchemists in seeking to make gold endeavored to transform this belief into tangible reality. . . . Now the discovery of constituents common to all atoms brings us anew to the conception of the Greeks and to the Unity of Matter." He suggests that his magneton may be one of the constituent elementary magnets which Ritz imagines arranged in neat rows with electrons revolving about them in order to explain the orderly series of spectral lines emitted by the atom.

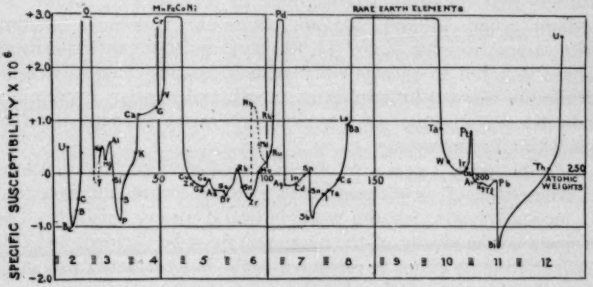
It is a logical sequence which leads from the definition of the magneton as a quantity to its recognition as a separate entity. A study of the weight of Philadelphia chimneys would show that all the weights have a greatest common divisor, but it would not be long before the investigator would conclude that this particular weight is associated with some individual piece of matter, and the conception of a brick would be forthcoming.

Can the magnitude of the magneton be accounted for by supposing it due to one electron revolving in an atom? The magnetic moment of such an electron orbit can be calculated from a knowledge of three other quantities: the electric charge of the electron, the area of its orbit, and its time of revolution. When these are substituted in the formula for the moment a quantity is obtained equal to about one-tenth of the magneton. It is significant that they are not very different in magnitude.

Gans and Weiss have attacked magnetic problems from different sides. Weiss postulated a molecular field and arrived at the conception of the magneton. Gans, on the other hand, starts from a study of elementary magnets and their groupings and deduces a law for the susceptibility of ferromagnetic substances which is equivalent to that found by Weiss. It strengthens our confidence in this law that it has been derived independently by two different investigators following diverse methods.

Much interest attaches itself to investigations of magnetic properties at low temperature because the motion due to heat plays a less and less important part as the absolute zero is approached. The idea had occurred that those three metals, vanadium, chromium, and manganese, which lie next to the strongly magnetic group, iron, nickel, and cobalt, might, like these, be really magnetic, but have their Curie point so far below ordinary temperatures that they behave as paramagnetic, just as iron acts at high temperature. Onnes and Weiss investigated this possibility, but they found no trace of ferromagnetism in vanadium and chromium. Manganese was paramagnetic, but after having been fused it was ferromagnetic.

FIG. 8.



Gadolinum sulphate faithfully followed Curie's law that specific susceptibility is inversely as the absolute temperature even down to 17° absolute. At the melting-point of hydrogen it deviates slightly. At 290.3° absolute its specific susceptibility is proportional to 73, while at 13.91° absolute it has grown to 1468.

Oxygen is of peculiar importance for magnetic investigations, because as a gas it is paramagnetic, while most other gases are diamagnetic, and, besides, it can be worked with in the solid, the liquid, and the gaseous state. Curie showed that from 20° C. to 452° C. gaseous oxygen follows his law. Onnes and Oosterhuis found that it continues to do this down to about 170° absolute. Liquid oxygen does not follow Curie's law that the product of absolute temperature and specific susceptibility is constant. It does, however, admit the relation that the square root of the

absolute temperature multiplied by the susceptibility is constant. This relation holds from 90° to 65° absolute. Solid oxygen has a much lower susceptibility than the liquid. When the liquid freezes the susceptibility drops to about one-third, then as the temperatures reaches – 240° C. another drop to one-half occurs in the susceptibility.

Any explanation of magnetic phenomena at low temperatures is complicated by the approach of the molecules to each other and by our ignorance as to whether any energy remains in matter at the absolute zero.

Let me call your attention to a curve showing the connection between specific susceptibility and atomic weight. Notice the periodic change from positive to negative susceptibility, the change, that is, from paramagnetic to diamagnetic. Take the sodium group, lithium, sodium, potassium, rubidium, cæsium, with atomic weights 7, 23, 39, 85, 133, and note their position. There is a fall in susceptibility with increasing atomic weight. While the curve is by no means smooth and regular, it certainly indicates a connection of a periodic character between the two quantities it represents.

At this point I cease my discussion of modern investigations in magnetism. It is not possible to give any impression save that of incompleteness, for the work is by no means finished. The problem of the reality of the magneton must be decided, and this conception must either be relegated to the limbo where dwell such cast-off terms as caloric, the N rays, hot ice, and the emission theory of light, or, with the electron, the alpha, beta, and gamma rays, be accepted as members in good standing of physical society. Who would want to serve a science that is finely rounded and complete, with no questions lacking an answer, with no phenomena waiting for explanation? Such a science might be beautiful, symmetrical, logically perfect, but it would also be dead.

(NOTE.—The writer desires to make his acknowledgments to a number of authors for diagrams and other material employed in the above paper.)

NORTHEAST HIGH SCHOOL,
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STANDARDIZED COLORED FLUIDS.[1]

BY

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AND

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THIS is the fifth of a series of papers [2] on the subject of color standardization, the work being begun by one of us in connection with his duty as chairman of a sub-committee of the Committee on Revision of the National Formulary, to which was entrusted the task of obtaining reliable and definite standards for the various coloring agents—notably cudbear and caramel—for which recipes are provided in that book. In pursuing this investigation, there was devised a plan of color standardization which afforded a simple and practical way of establishing color standards.

As fully explained in the paper read before the Eighth International Congress of Applied Chemistry, the idea of preparing any shade of color by blending permanent standardized colored fluids of red, yellow, and blue appealed to him; and, as reported at that congress, the first set of such colored fluids was prepared by making half-normal solutions of cobalt, ferric iron, and copper. From these solutions were prepared the 88 blends, representing all the possible combinations in a total volume of 12 Cc. of fluid, where the amount of each of the three ingredients ranged from 1 to 12 Cc., fractional parts of the cubic centimetre being avoided.

The idea of using solutions of colored metallic salts for matching tints has already been employed in water analysis by Crookes, Odling and Tidy, and by Hazen; but the plan mentioned above has the following points of originality:

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[1] Communicated by Dr. Arny.

[2] "Color Standards," by H. V. Arny, *American Druggist*, 60 (1912), 35. "International Standards for Colored Fluids," by H. V. Arny, *Proceedings of the Eighth International Congress of Applied Chemistry* (1912), 26, 319. "Weiteres über Normal Farben," by H. V. Arny, *Deutsch-Amerikanische Apotheker Zeitung*, 33 (1913), 165. "The Problem of Color Standardization," by H. V. Arny and E. G. Pickhardt, *Druggists' Circular*, 57 (1914), 131.

First.—The cobalt, iron, and copper solutions are prepared as definite volumetric solutions of half-normal strength.

Second.—The blending of these in any proportion to a constant volume insures the widest possible range of tints.

It was found that in blends of the *neutral* (unacidulated) half-normal solution precipitation occurred within a few days after preparation; so, in subsequent work, *acidulated* solutions of half-normal strength were used, the solvent being a diluted hydrochloric acid made from 25 Cc. of concentrated (30 per cent.) acid and 975 Cc. of water.[3] This modified the tints of the original solutions and the blends, but not enough to seriously affect the value of the fluids as standard colors, which were thus made practically permanent.

This line of colors, which we called the "*Co-Fe-Cu*" tints, was fully described in the *Druggists' Circular* for March, 1914, and in that article it was frankly stated that the value of these tints was lessened through the fact that the cobalt solution did not furnish a sufficiently pure red color. In short, while the "Co-Fe-Cu" blends furnished an ideal line of green, yellow, and orange tints, they lacked in deep reds and dark blues.

During the past year we have been able to pursue the problem to a successful issue by preparing a line of ammoniacal fluids which furnish most of the necessary tints.

This line, which we call the "Co-Cro-Cu" fluids, was made up as follows:

Red tenth-normal ammoniacal cobalt solution was prepared by dissolving 13.5 grammes of roseo-cobaltic-chloride ($CoCl_3$ $5N$-$H_3.H_2O$) in enough of a fluid consisting of 100 Cc. of 28 per cent. ammonia water and 900 Cc. of water to make a litre of finished product.[4]

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[3] Since this strength of acid was used throughout our work, since it was made up empirically, and since its approximate strength is about 1 in 100, we will in future speak of it as 1 per cent. hydrochloric acid.

[4] The tenth-normal and fiftieth-normal solutions here described are based on the absolute amount of metal contained in each. As cobalt has the atomic weight 58.97 and a valence of 2, we assume that a tenth-normal cobalt solution will contain 2.9485 Gm. of the metal per litre and that the fiftieth-normal will contain one-fifth of this amount. As copper has the atomic weight 63.57 and the valence is 2, we assume that a tenth-normal copper solution will contain 3.1785 Gm. of the metal to the litre and that the fiftieth-normal will contain one fifth of this amount. As chromium has the atomic weight 52.00

The solution thus prepared was then assayed gravimetrically and so diluted that each cubic centimetre of fluid contained 0.0029-485 Gm. of metallic cobalt. Experience showed, however, that empiric solutions prepared by dissolving 13.43 Gm. of the roseo-cobaltic chloride in a diluted ammonia of the strength just stated did not vary sufficiently from the standard in color intensity to make assay necessary in average work.

If the commercial roseo-cobaltic chloride is not obtainable, it can be prepared as follows: Dissolve 12 Gm. of cobalt chloride ($CoCl_26H_2O$) in about 100 Cc. of water. After filtration, add 100 Cc. of 28 per cent. ammonia, shake well, and keep in a warm place (between 50° to 70° C.) six or eight hours, with occasional shaking. Let stand over night at ordinary room temperature. Warm again at same temperature as before and gradually dilute with water, a small portion at a time in such a way that the temperature of the liquid will not go below 50°, until it has been diluted to 1 litre. We now have an empiric tenth-normal ammoniacal cobalt chloride of a bright red color. This solution shows a yellow tinge when examined closely and therefore cannot be used as a standard. But from it the roseo-cobaltic salt can be made by the addition of an excess of concentrated hydrochloric acid and then boiling for several minutes until a precipitate forms. The precipitation can be hastened by chilling with running water. The precipitate collected on a filter is washed with 1 per cent. hydrochloric acid and is then dried in oven at about 100° C.

Yellow tenth-normal ammoniacal chromium solution was made by dissolving 3 Gm. of ammonium dichromate ($(NH_4)_2Cr_2O_7$) in 50 Cc. of 1 per cent. hydrochloric acid, then adding 50 Cc. of 28 per cent. ammonia water, and finally diluting with enough water to make one litre. The solution was then assayed gravimetrically and so diluted that each cubic centimetre of fluid contained 0.0008666 Gm. of chromium. It was found, however, that by making an empiric solution of 2.1 Gm. of ammonium dichromate

and presumably has the valence 6 in the dichromate, then a tenth-normal solution will contain 0.8666 Gm. of chromium to the litre and a fiftieth-normal will contain one-fifth of this amount. This conception of fiftieth-normal chromium is quite different from the commonly-accepted term, tenth fiftieth or hundredth-normal dichromate solution used in the oxidizing titrations; in which case, as is well known, the titre of the solution is based on its oxygen-producing capacity, rather than its absolute chromium content.

dissolved in 1 per cent. hydrochloric acid and then adding 28 per cent. ammonia water and diluting with water to 1 litre the color value was the same as that of the standardized solution.

Blue tenth-normal ammoniacal copper solution was prepared by diluting 200 Cc. of standard assayed half-normal copper sulphate (containing 62.43 Gm. of $CuSO_45H_2O$ to the litre) to tenth-normal alkaline fluid, by adding 100 Cc. of 28 per cent. ammonia water and then making up to 1 litre with water. Each cubic centimetre of this solution contained 0.0031785 Gm. of metallic copper.

As with the other fluids, a carefully-prepared empiric solution gave the same color value as the assayed standard.

Assayed standard half-normal solutions of both copper sulphate and copper chloride were prepared. Each was diluted to tenth-normal ammoniacal copper in the manner just outlined and the two blue colors compared. They were practically identical both in tint and intensity of color. In making empiric copper solution, however, only perfect crystals of chemically pure copper sulphate were used.

Further data concerning these three standard solutions will be found below. At this place we will merely point out that for ordinary purposes of comparison we found the blends prepared from these tenth-normal ammoniacal solutions were too dark. We therefore diluted them down with water to equal fiftieth-normal solutions, and from these latter we prepared the " Co-Cro-Cu " blends.

COMPARISON OF STANDARD TINTS.

When starting on the investigation of which this paper is a part, the first difficulty encountered was the lack of an accurate and available basis of comparison for fluid colors. Of course, the solar spectrum itself was the first standard thought of, and one of us, with Dr. H. H. Schaefer, spent some weeks during 1913 attempting to match our colored fluids against this natural standard. But like other investigators, we found that the very brilliancy of the spectrum colors was a drawback, and that our colored fluids, attractive though they were, paled before the glory of a spectrum segment. We still hope, however, that we may yet be able to devise some scheme whereby our colors can be expressed in terms of solar tints.

Finding, for the time being, this ideal plan futile, we turned to that interesting color apparatus, the Lovibond tintometer, which, as was explained in the *Druggists' Circular* article cited above, possessed the advantages of simplicity, permanency, and availability. Another of its advantages for our work was that the colors of the glass slides used in the tintometer were not superior to the tints of our standard colored fluids.

In the article just referred to, the Lovibond readings of the acidulated half-normal (red) cobalt, (yellow) ferric iron, and (blue) copper solutions were given, as well as similar readings of the 88 blends already mentioned.

During the past winter Lovibond readings of the "Co-Cro-Cu" blends have been made, and the results are tabulated below:

LOVIBOND READINGS OF THE TENTH-NORMAL STANDARD AMMONIACAL SOLUTIONS IN A ¼-INCH CELL.

NOTE.—In this and all other Lovibond readings in this paper, the red Lovibond glasses were No. 200, the yellow were No. 510, and the blue were 1180.

Roseo-Cobaltic Chloride ...	Red = 5.0 + 4.0 + 3.0 + 2.0 + 1.9 + 1.8 + 1.6 = 19.3 Yellow = −0.7 Blue = −1.6
Ammonium Dichromate ...	Red = −0.3 Yellow = 5.0 + 4.0 + 0.4 = 9.4 Blue = None
Copper Sulphate	Red = −4.0 Yellow = None Blue = 5.0 + 4.0 + 3.0 + 2.0 + 1.9 + 1.3 = 17.2

FIFTIETH-NORMAL STANDARD AMMONIACAL SOLUTION IN ½-INCH CELL.

Roseo-Cobaltic Chloride ...	Red = 5.0 + 3.0 + 0.2 = 8.2 Yellow = None Blue = −0.4
Ammonium Dichromate ...	Red = None Yellow = 5.0 + 1.0 + 0.4 = 6.4 Blue = + 0.2
Copper Sulphate	Red = None Yellow = None Blue = 5.0 + 1.0 + 0.4 = 6.4

TOTAL LOVIBOND READINGS OF THE FIFTIETH-NORMAL STANDARD AMMONIACAL "CO-CRO-CU" BLENDS IN ½-INCH CELL.

Co-Cro-Cu	Red	Yellow	Blue
12 - 0 - 0	5.0 + 3.0 + 0.2	None	−0.4
11 - 1 - 0	5.0 + 1.9	0.7	−0.4
10 - 2 - 0	5.0 + 1.5	1.5	−0.4
9 - 3 - 0	5.0 + 0.7	2.0 + 0.4	−0.3
8 - 4 - 0	5.0 + 0.1	3.0	−0.3
7 - 5 - 0	3.0 + 1.5	2.0 + 1.6	−0.3
6 - 6 - 0	4.0	4.0	−0.2
5 - 7 - 0	3.0	4.0 + 0.2	−0.2
4 - 8 - 0	2.0	4.0 + 0.4	−0.2
3 - 9 - 0	1.0 + 0.4 + 0.1	5.0	−0.2
2 - 10 - 0	0.7 + 0.2	5.0 + 0.4	−0.1
1 - 11 - 0	0.4	6.0	None
0 - 12 - 0	None	5.0 + 1.0 + 0.4	+ 0.2
0 - 11 - 1	None	4.0 + 1.6	0.4 + 0.1
0 - 10 - 2	None	5.0	1.0 + 0.2
0 - 9 - 3	None	4.0 + 0.2	1.0 + 0.4
0 - 8 - 4	None	3.0 + 0.7 + 0.1	2.0
0 - 7 - 5	None	3.0 + 0.2	2.0 + 0.2 + 0.1
0 - 6 - 6	None	3.0	3.0 + 0.2
0 - 5 - 7	None	2.0 + 0.2 + 0.1	3.0 + 0.4 + 0.2
0 - 4 - 8	None	1.9	4.0 + 0.1
0 - 3 - 9	None	1.6	4.0 + 0.7
0 - 2 - 10	None	1.0	5.0 + 0.2
0 - 1 - 11	None	0.4 + 0.1	5.0 + 0.7
0 - 0 - 12	None	None	5.0 + 1.0 + 0.4
1 - 0 - 11	0.7	None	5.0 + 0.7
2 - 0 - 10	1.3 + 0.1	None	5.0
3 - 0 - 9	2.0 + 0.1	None	4.0 + 0.2
4 - 0 - 8	2.0 + 0.7	None	3.0 + 0.7
5 - 0 - 7	3.0 + 0.4 + 0.2	None	3.0 + 0.2
6 - 0 - 6	4.0 + 0.2	None	2.0 + 0.4
7 - 0 - 5	3.0 + 1.9	None	1.8
8 - 0 - 4	5.0 + 0.7	None	1.3 + 0.1
9 - 0 - 3	5.0 + 1.3	None	0.7 + 0.1
10 - 0 - 2	5.0 + 1.6	None	0.2 + 0.1
11 - 0 - 1	5.0 + 2.0 + 0.3	None	−0.1
10 - 1 - 1	5.0 + 1.0	0.7	None
9 - 2 - 1	5.0 + 0.7	1.5	None
8 - 3 - 1	5.0	2.0 + 0.1	None
7 - 4 - 1	4.0 + 0.2 + 0.1	2.0 + 0.2	None
6 - 5 - 1	3.0 + 0.4 + 0.1	1.5 + 1.0	None
5 - 6 - 1	3.0	3.0	None
4 - 7 - 1	2.0	2.0 + 1.5 + 0.1	None
3 - 8 - 1	1.6	4.0 + 0.1	None

Co-Cro-Cu	Red	Yellow	Blue
2- 9- 1	0.7 + 0.2	4.0 + 0.7	None
1-10- 1	0.2	5.0 + 0.2 + 0.1	None
1- 9- 2	None	4.0 + 0.4	+ 0.7
1- 8- 3	None	2.0 + 1.6	1.3
1- 7- 4	None	3.0 + 0.2 + 0.1	1.8
1- 6- 5	None	1.6 + 1.0	2.0 + 0.4
1- 5- 6	None	2.0 + 0.2 + 0.1	3.0
1- 4- 7	None	2.0	2.0 + 1.8
1- 3- 8	None	1.5	3.0 + 1.3
1- 2- 9	None	1.0 + 0.1	3.0 + 1.9
1- 1-10	None	0.7	5.0 + 0.4 + 0.1
2- 1- 9	0.2	0.4	4.0 + 0.7
3- 1- 8	1.0	0.2 + 0.1	3.0 + 0.4 + 0.2
4- 1- 7	1.9	0.2 + 0.1	3.0
5- 1- 6	2.0 + 0.4	0.4	2.0 + 0.1
6- 1- 5	3.0 + 0.4	0.4	1.6 + 0.1
7- 1- 4	4.0 + 0.1	0.4	1.0 + 0.2
8- 1- 3	5.0	0.4	0.7
9- 1- 2	5.0 + 0.7	0.4	0.1
8- 2- 2	3.0 + 1.6	0.7	0.1
7- 3- 2	4.0 + 0.1	1.3 + 0.1	0.1
6- 4- 2	3.0 + 0.4	1.5	0.2
5- 5- 2	2.0 +0.2	2.0 + 0.2 + 0.1	0.1
4- 6- 2	1.9	1.8 + 1.0	0.4 + 0.1
3- 7- 2	1.0 + 0.1	3.0 + 0.1	0.2
2- 8- 2	0.4	2.0 + 1.5	0.4
2- 7- 3	0.4 + 0.1	3.0	1.0
2- 6- 4	0.2	1.0 + 1.3	1.3
2- 5- 5	0.2	2.0 + 0.1	2.0 + 0.1
2- 4- 6	0.2	1.6	1.6 + 1.0 + 0.1
2- 3- 7	0.2	1.0 + 0.1	3.0 + 0.2
2- 2- 8	0.2	0.7	2.0 + 1.8
3- 2- 7	1.0 + 0.1	0.7	3.0
4- 2- 6	1.9	0.7	1.5 + 1.0
5- 2- 5	2.0 + 0.4	1.0	1.6 + 0.1
6- 2- 4	3.0 + 0.4	0.7 + 0.1	1.5
7- 2- 3	4.0 + 0.1	0.7	0.4 + 0.1
6- 3- 3	3.0 + 0.4	1.3	0.7 + 0.1
5- 4- 3	2.0 + 0.2	1.9	1.0
4- 5- 3	1.9	2.0 + 0.2 + 0.1	1.0
3- 6- 3	1.0	2.0 + 0.4 + 0.2	0.7
3- 5- 4	1.0	2.0 + 0.2	1.0 + 0.4
3- 4- 5	1.0	1.6	1.9
3- 3- 6	1.0	1.0 + 0.1	2.0 + 0.2
4- 3- 5	1.8	1.3 + 0.1	2.0 + 0.1
5- 3- 4	2.0 + 0.4	1.5	1.0 + 0.2 + 0.1
4- 4- 4	1.9	1.9	1.8

INTERPRETATION OF THE TABLES.

The foregoing tables give only a glimpse of the many interesting characteristics of these fluids and their blends.

In the first place, it is necessary to call attention to the fact that the glasses from which the Lovibond readings were obtained are so graduated that the color intensity of any red glass of a given number approximates the color intensity of a yellow glass or of a blue glass of that same number. This agreement is based on the fact that when three glasses, red, yellow, and blue, each having the same number, are superimposed and held to the eye in a good light, with a perfectly white perspective, a neutral gray tint is the result.

The numbers on the glasses represent units of color value. Thus the glass labelled "0.1" is supposed to have only one-tenth the color intensity of the glass labelled "1.0," and therefore ten "0.1" glasses should match one "1.0" glass in color intensity. This principle holds good for all practical purposes when not more than three or four glasses are used at one time; but if ten "0.1" glasses, say of the yellow, were compared with one "1.0" yellow glass, the ten glasses would show a much darker effect, owing to the fact that there is ten times the thickness of plain glass to look through.

There are some blends of the glasses that do not match perfectly with other blends of supposedly the same total Lovibond value, even when each blend is composed of the same number of glasses, but these differences are very slight.

We have found that a reading, say Blue = 5.0 + 1.0 + 0.4, can, for all practical purposes, be considered as meaning 6.4 Lovibond color units, and can be matched, fairly closely, by any other combination of blue glasses that total 6.4 units. In the light of this statement it is interesting to note in the above table the following facts concerning the basic fiftieth-normal fluids:

The red fiftieth-normal ammoniacal cobalt solution shows a Lovibond value totalling 8.2 units of red, no yellow, and −0.4 unit of blue.

The yellow fiftieth-normal ammoniacal chromate solution shows a Lovibond value totalling 6.4 units of yellow, no red, and 0.2 unit of blue.

The blue fiftieth-normal ammoniacal copper solution shows a Lovibond value totalling 6.4 units of blue, no red, and no yellow.

In the summary just given, and also in the tables, we find

minus values. Thus the fiftieth-normal cobalt solution was so much purer red than the Lovibond glasses that, to secure a perfect match in tint, it was necessary to place 0.4 unit of blue glass in front of the cell containing the fluid when making the test.

The figures just given show that the total net color intensity of the three basic solutions—the red, the yellow, and the blue—is nearly equal when expressed in Lovibond units. Then, even as three Lovibond glasses, each of the same color value of red, yellow, and blue, give a neutral gray tint when combined, so equal volumes of the three fiftieth-normal ammoniacal color fluids should likewise give a neutral gray or almost colorless fluid when combined. This proves to be the case, with but a slight variation, owing to the fact that the cobalt standard solution has a little higher color value than either of the other two.

Still more interesting is the sequence of colors obtained when these three fiftieth-normal ammoniacal solutions are blended systematically. When each blend was put into a one-ounce Blake bottle, and the entire 88 blends, with a sample of each of the three standard solutions, were sorted by the eye, they fell naturally into five groups.

The first group, consisting of 36 samples, are pure spectrum colors, and represent the three fiftieth-normal standard colors, together with all the possible blends obtained by mixing any two of the three standards in cubic centimetre lots to a total volume of 12 Cc.

The second group consists of 27 hues, which are a trifle less bright than the first group. These represent all the possible blends obtained by mixing any two of the three standard fluids in cubic centimetre lots to equal a total volume of 11 Cc. and then adding to each blend 1 cubic centimetre of the odd standard fluid, making the entire volume of each blend equal 12 Cc.

The third group, consisting of 18 blends, is still less brilliant, and the blends here suggest a nearness to the neutral tint. They represent all the possible combinations obtained by mixing any two of the three standards in cubic centimetre lots to equal a total of 10 Cc. and then adding to each blend 2 Cc. of the odd standard, making the entire volume of each blend equal 12 Cc.

The fourth group, consisting of nine blends, is practically a group of neutral tints. These blends represent all the possible combinations obtained by mixing any two of the three standards in

cubic centimetre lots to equal a total of 9 Cc. and then adding to each blend 3 Cc. of the odd standard, making the entire volume of each blend equal 12 Cc. Although the blends in this group are so near to neutral there is, in nearly each one, a sufficient preponderance of one of the three primary colors to give it its proper place in the system as a "neutral red," "neutral yellow," or a "neutral blue."

The fifth group consists of only one blend and contains an equal volume of each of the three fiftieth-normal ammoniacal standards. Theoretically this should be a perfect neutral gray, but owing to the fact that the standard red cobalt solution has a little greater color intensity than either the yellow or the blue, as will be seen in the table of Lovibond readings, this last blend of equal volumes shows a slight excess of red when observed closely.

When these five groups of 91 tints are arranged in a series of graduated circles, one inside another, in the mathematical order in which they appear in the table, an automatic sub-grouping of color effects that is produced is very interesting. The first group is the outer circle, and the blends are arranged from left to right in the same order that colors appear in the solar spectrum. Starting with blend "12 – 0 – 0," which is the pure red fiftieth-normal cobalt solution, we go from this through orange to yellow, then through green to blue, and then through violet back to red, blend "11 – 0 – 1" completing the first circle.

The second group of 27 blends is then arranged just inside the first circle, starting with blend "10 – 1 – 1," which takes its place under "12 – 0 – 0," and the circle is completed with blend "9 – 1 – 2."

The third group of 18 blends is now arranged inside of the second circle, starting with blend "8 – 2 – 2," which takes its place under "10 – 1 – 1," and the circle is completed with blend "7 – 2 – 3."

The fourth group is arranged in the same way inside of the third circle, blend "6 – 3 – 3" being placed under "8 – 2 – 2," and the circle ends with blend "5 – 3 – 4."

The fifth group, which is represented by only the one neutral equal volume blend, is now placed in the centre and forms what we may call the hub of the wheel. If this circular plat is appropriately subdivided into nine equal (40°) segments, we find that each segment contains a color group in itself.

THE PERMANGANATE-DICHROMATE BLENDS.

In reporting on the "Co-Fe-Cu" colored fluids and blends the limitations of the range of tints was frankly stated. The preparation of the ammoniacal line of "Co-Cro-Cu" fluids furnished us with the means of producing nearly every tint necessary to our needs. However, the pink, cardinal red, and a peculiar quality of violet color were not forthcoming.

As before stated, the "Co-Fe-Cu" fluids furnished practically every needed shade of orange, yellow, and green, while the "Co-Cro-Cu" line furnished the necessary blends in blue, bluish-red, and reddish-blue.

We have found that most of the tints which could not be obtained with either the "Co-Fe-Cu" or the "Co-Cro-Cu" line of colored fluids can be produced by blending proper proportions of a solution of potassium permanganate with a solution of potassium dichromate.

The permanganate solution is prepared by diluting the usual assayed tenth-normal volumetric solution to one-thousandth-normal strength so that each cubic centimetre contains 0.00001 Gm. of the element manganese. The yellow dichromate is prepared by diluting the usual assayed tenth-normal volumetric solution to one-hundredth-normal strength,[5] so that each cubic centimetre contains 0.00017 Gm. of the element chromium.

The Lovibond readings of these two standard solutions are here given:

ONE-THOUSANDTH-NORMAL POTASSIUM PERMANGANATE IN ½-INCH CELL.

Red	Yellow	Blue
5.0 + 1.3	−0.7	+ 0.2

ONE-HUNDREDTH-NORMAL POTASSIUM DICHROMATE IN ½-INCH CELL.

Red	Yellow	Blue
0.3	5.0 + 2.0 + 0.1	− 1.0

These readings show that the intimation conveyed in the Chemical Congress article of the likelihood that equimolecular solutions of the colored fluids when blended in equal volumes would give the neutral gray tint is not confirmed by our experiments except in the case of the ammoniacal fluids.

[5] See footnote 4, p. 200.

THE SCOPE OF THE STANDARD COLORED FLUIDS.

It is hoped that the testimony thus far produced shows beyond cavil the fact that practically every tint of color as manifested in fluids can be duplicated by the proper blending of a few standard colored fluids.

We have shown that if the analyst adds to his stock of reagents and volumetric solutions five other assayed fluids, namely, acidulated half-normal cobalt chloride, acidulated half-normal ferric chloride, acidulated half-normal copper sulphate or copper chloride, ammoniacal tenth-normal roseo-cobaltic chloride, and ammoniacal tenth-normal ammonium dichromate, he will have all the essential ingredients from which, by blending or diluting, he can prepare any desired hue.

Of the eight standard fluids recommended as blending ingredients, the "Co-Fe-Cu" fluids are provided for in the list just given. The standard ammoniacal fiftieth-normal chromate and cobalt fluids are prepared from the tenth-normal solutions by proper dilution, while the ammoniacal fiftieth-normal copper solution can be prepared directly from the standard half-normal copper solution. The hundredth-normal dichromate and the thousandth-normal permanganate are prepared by diluting the regular U. S. P. standard volumetric solutions that every well-equipped laboratory keeps on hand.

As to the keeping qualities of the standard colored fluids and their blends, we showed in the *Druggists' Circular* article that the "Co-Fe-Cu" solutions and blends were practically permanent. Lovibond readings of samples a year old were approximately the same as readings made when they were freshly prepared. Samples of the blends that are now three years old show some color difference, but it seems reasonable to consider as stable a reagent that remains unchanged for more than a year.

As to the ammoniacal "Co-Cro-Cu" solutions, the standard cobalt and chromate fluids are practically permanent for six months or even a year if they are kept in well-stoppered bottles. The ammoniacal copper solution shows precipitation within a few weeks after preparation, its stability depending on the amount of free ammonia in it and also on how tightly closed the bottle is kept. As it is a very simple operation to prepare it freshly from the half-normal standard acidulated copper solution, this should be done when it is needed for accurate work.

The permanganate and dichromate solutions, while stable when kept separately, are not permanent when mixed. Blends of these two are not dependable for more than an hour or so after mixing, and consequently should be freshly prepared when needed.

THE SIGNIFICANCE OF THE STANDARD COLORED FLUIDS.

Having shown that practically every hue can be produced by proper blending of the eight standard colored fluids—the "Co-Fe-Cu," the "Co-Cro-Cu" and the permanganate-dichromate solutions—it is pertinent to ask, "Of what importance are these fluids and their blends?"

In the first place, we believe that further study of these fluids will throw much light on the problem of the relationship between color and the molecular structure. We might point out the curious fact that, barring the platinum group, the only metals that provide colored ions are the six whose atomic weight are in sequence.

Chromium Atomic weight	Manganese Atomic weight	Iron Atomic weight	Nickel Atomic weight	Cobalt Atomic weight	Copper Atomic weight
52	54.93	55.84	58.68	58.97	63.57

What relationship exists between the color and the other physical data concerning these fluids further experimentation alone will show. The immediate importance of these standard colored fluids and their blends is demonstrated by the question of practical color standardization which started the investigation, of which this paper is the fifth contribution.

There is great need for a simple and practical standardization of colors that will have a scientific rather than empiric basis. In music we have practical standards of sound pitch in the air column and vibrations of the tuning-fork, the latter being a practical appliance which is understood even by the unscientific mind. In color the only definite standard is the exceedingly difficult measurement of the length and the speed of the vibrations of light waves, and up to the present time not only have all efforts to gauge colors been empiric, but in many cases the color standards, such as lithographed plates and dyed fabrics, have easily faded on exposure to light.

The color standards described in this article have advantages over any other hitherto suggested in the following way:

1. They are scientifically exact, since based on the molecular weights of their constituent salts.

2. They are simple and easily duplicated, since they are prepared and standardized by methods that can be carried out without difficulty by any chemist.

3. Of the eight standard fluids, the five that should be assayed are more stable than any other proposed color standard except glass; the other three can be prepared in a few minutes from stable solutions, two of which are always a part of the stock in every well-equipped chemical laboratory.

4. While these colored fluids may not be as stable as colored glass, they possess marked advantages over that material, in that they are easily prepared and quickly duplicated.

5. The device of preparing blends by admixture of the three primary colored fluids in any proportion desired by the operator permits the production of infinitesimal differences in shades, while the fact that they are fluids miscible with water allows any intensity of color that may be desired.

6. While the work so far is concerned only with colored fluids, and while it is not a simple matter to match an opaque reflected color with a transparent one, our preliminary experiments in that line have shown us that the ultimate solution of that phase of the problem is merely a matter of further work.

PRACTICAL APPLICATION OF THE COLORED FLUIDS.

Extending the question "Of what importance are these colored fluids and their blends?" to the field of every-day work, the best answer we can give is that during the past winter we have studied several colorimetric tests and have found matches for their tints in blends of our standard fluids. The tests which we have so far studied in the above connection are as follows:

1. Nitrogen as ammonia with Nessler's reagent.
2. Nitrogen as nitrate with phenol-sulphonic acid.
3. Nitrogen as nitrite with naphthylamine and sulphanilic acid.
4. Vanillin with bromine water and ferrous sulphate.
5. Vanillin with phosphotungstic-phosphomolybdic acid reagent.
6. Uric acid with phosphomolybdic acid.
7. Phosphorus pentoxide with ammonium molybdate.

8. Salicylic acid with ferric chloride solution.

The first five in this list were given some attention in the *Druggists' Circular* article already referred to, but our recent work on them has been of a much more exhaustive nature. The rest of the tests mentioned have been corroborations of various authorities in so far as merely chemical tests are concerned; but the process of matching them against our standard fluid blends has been work in a virgin field. The practical results of all the nine tests will be taken up in detail in another paper.

COLUMBIA UNIVERSITY COLLEGE OF PHARMACY,
June, 1915.

Life-testing Equipment for Tungsten Lamps. ANON. (*Electrical World*, vol. 66, No. 1.)—Very few lines of work require voltage to be held within the narrow limits of one-tenth of one per cent., but in the accurate testing of incandescent lamps potential precision is regarded as a prime essential. Therefore, in planning the new life-testing equipment for the National Lamp Works of the General Electric Company at Mela Park, Cleveland, precise voltage regulation and economical construction were the guiding factors in an installation embodying many unique and original ideas.

As is well known, the tungsten lamp is sensitive to variations of voltage, and to obtain a good and uniform life it is necessary, even in a commercial way, to keep the voltage well under control. The fundamental part of lamp-life testing is the preservation of accurately uniform voltage at the testing racks. The pains taken to secure this are described in detail.

The Pennsylvania Railroad Test Department. C. D. YOUNG. (*Railway Age Gazette*, vol. 89, No. 7.)—The indifference of American manufacturers to the benefits of maintaining adequate laboratories of inspection and research, as compared with the activity in that field of foreign industrial organizations, has been a matter of frequent comment. A notable exception to this rule will be found in the description of the splendid development of the Pennsylvania Railroad Test Department in a paper read at the annual meeting of the American Society for Testing Materials, held at Atlantic City, June 22 to 26, 1915. From its humble beginnings in 1874 the work of the department has increased at an average rate of over 100 per cent. in each year, till the number of routine physical tests conducted reached, in 1912, a total of 120,000, the carrying out of which required the services of 300 employees. Although the cost of operating this department for one year, $534,000, seems enormous, it amounts to only 0.6 per cent. of the material inspected and tested.

Plating by Impact. ANON. (*Scientific American Supplement*, vol. lxxxix, No. 2060.)—The Schoop process of plating by means of a metal spray is now well known and in commercial use. A process of the same type has been in course of development by C. F. Jenkins, of Washington, D. C. This process can best be understood if it is remembered that when an electric lamp bulb gives way a discoloration of the inside of the bulb occurs, and also that when a fuse plug "blows" the mica cover is discolored. This color is black when the fuse is of lead, but it is a reddish color when a piece of copper wire is used. This would seem to indicate some kind of deposit resulting from the blowing of the fuse, that it is not completely volatilized. Under a magnification of 300 diameters or more, minute particles of the copper wire are discovered adhering to the cover of the fuse plug, and, when a common visiting card is used for a cover instead of the mica, a decided deposit is attained. Repeated charges of such a fuse result in a complete coating of the card. When this surface is burnished with some smooth, hard object, a shiny polished metal surface results. A fuse used in this manner is, in effect, a gun which throws out a shower of miniature shot so small as to be invisible to the naked eye, and this would operate successfully for covering almost any surface but for the annoyance of the frequent replacements with short pieces of copper wire. This led naturally to the development of a special "gun" into the barrel of which a copper wire is continuously fed. A pair of small rollers actuated by a motor pulls the wire off the supply spool and projects it across the barrel until the end touches the opposite surface. The inner lining of the barrel and the propelling rollers form a short circuit. The wire is immediately melted, and the heat causes it to be thrown out of the barrel against any object held in position for that purpose. When a plurality of wires or a flat ribbon is used in order to cover a larger area in a given time, it is found desirable to add a propelling force, and this is done by introducing into the barrel behind the wire a small charge of explosive gas. The melting of the wire explodes the charge which projects the miniature metallic particles that are momentarily suspended in the gas against the object to be coated. It has been found that objects in great variety can be coated in this manner, and that any electrically conductive material can be used for the purpose.

Agate and Onyx. Press Bulletin. (*U. S. Geological Survey*, June–July, 1915, No. 223.)—The distinction between agate and onyx is not apparent to every one, as indicated by the samples of the two minerals received by the United States Geological Survey with requests for information. Onyx marble or Mexican onyx is composed of calcium carbonate or banded limestone. True agate is a variety of silica. Onyx marble is much softer than agate and is rarely used for gems, but when onyx is obtained in pieces of sufficient size it is cut and polished for small objects like inkstands and paper weights, as well as for table tops and soda fountains.

A FLICKER PHOTOMETER ATTACHMENT FOR THE LUMMER-BRODHUN CONTRAST PHOTOMETER.*

BY

E. F. KINGSBURY,

Physical Laboratory, The United Gas Improvement Company, Philadelphia.
Member of the Institute.

EQUALITY-of-brightness photometers and flicker photometers have heretofore been built entirely separately from one another. In the literature of the measurement of light the design and use of the one has been a distinct problem from the design and use of the other, though the results the former gives are often used as a criterion for the practicability of the latter. Consequently, in constructing a flicker photometer, it has been built from the beginning just for the one purpose, notwithstanding the fact that about half of a flicker photometer is identical with an equality-of-brightness photometer in its essential requirements. It should be possible, by interchanging the two dissimilar parts on the part common to both, to make either one easily converted into the other type of photometer. The result should be a photometer that is simple, compact, and capable of measuring either lights of one color or lights of widely different colors with the best precision of the equality-of-brightness method in the one case and of the flicker method in the other.

At the same time it is felt that such an instrument should embody the latest approved features that are coming to be recognized as desirable in a flicker photometer.

It is the purpose of this paper to describe such an instrument which the author has devised in the shape of a flicker attachment to replace the telescope of the ordinary Lummer-Brodhun contrast photometer when it is desired to measure differently colored lights. The high precision and the wide distribution of this type of equality-of-brightness photometer justify its selection for this purpose, although there are other good instruments to which this idea could be even more easily applied with a slight modification of details.

The various details concerning this attachment will be better appreciated if, before describing it in detail, we briefly review the essential requirements of a flicker photometer.

* Communicated by the Author.

Wilde [1] says the desiderata for an ideal photometer are:

1. It must be sensitive and certain on lights of different color.

2. Rays must fall upon the illuminated surface perpendicularly to reduce angle errors.

3. Illuminated surfaces should be directly over pointer on the bar.

4. When balance is nearly obtained, it should be easy to perceive which way the head must be moved to improve the balance, without the necessity of rocking it to and fro.

5. If complete reversibility is required, both illuminated surfaces should be identical and must be the same distance from the eye.

6. In a flicker photometer, in order to secure sensitiveness, the transition from one surface to the other must be sudden and there must be no dark line crossing the field of view, such as an unilluminated edge of a card.

These requirements are subject to considerable modification and addition. They are given, however, as illustrating a certain viewpoint in the construction and use of flicker photometers and to emphasize the employment of a proper photometric procedure.

The use of the substitution method of photometry is strongly recommended with the direct measurement of distances wherever necessary.

This makes the second, third, and fifth requirements unnecessary.

The fourth requirement is misleading, because one cannot obtain a satisfactory balance with the flicker photometer by merely looking at it in one position. The proper method of obtaining a setting is rapidly to vary the light intensity back and forth over the no-flicker peak, at the same time gradually narrowing the range until the point of minimum or no-flicker is found. The essential requirements only will be outlined below. They will not be discussed in detail, as they have been fully covered elsewhere.[2]

The sensitiveness or precision requirements of a flicker photometer depend fundamentally upon one thing—the focusing of the observer's maximum attention upon the elimination of intensity flicker. The attention is secured in a rather negative way by eliminating all external factors that can divert it.

[1] *London Illuminating Engineer*, p. 825, i, 1908.

[2] *Phil. Mag.*, July–December, 1912, pp. 149, 352, 744, 845.

First, undue tiring of the eyes should be guarded against. This is secured mainly by making the field surrounding the flickering field light instead of leaving it dark. The intensity of this illumination should be equal to, or less than, the intensity of the comparison field. If brighter, it attracts the attention. The intensity and color of this field do not seem to influence the accuracy, at least within wide limits. This field should be illuminated uniformly and should be free from mechanical defects.

Secondly, there should be no mechanical flicker in the comparison field itself. This is secured by keeping the optical system clean and making its focus in space and not on a surface. The focus of the eye-piece lens, of course, should be on the plane of the opening through the external field just referred to.

Thirdly, in securing a balance, the color flicker (as distinguished from the intensity flicker) should be reduced to a minimum by a proper regulation of the speed of alternation of the colors.

The requirements that make for agreement with the equality-of-brightness method are in each method:

First, a restricted field; and

Secondly, a high illumination on this field.

Dr. H. E. Ives has found that on a field size of about 2° and at an illumination of about 25-metre candles the flicker method and the equality-of-brightness method agree in their results, provided the same conditions are complied with in each case. A restricted field and a high illumination are somewhat interchangeable, but the candle-power of many standard lamps and other light sources places a limit on the illumination, so that the field is restricted to make it equivalent to a higher illumination.

The requirements having to do with procedure have been mentioned; namely, that the substitution method be used with the direct measurement of distances. Where it cannot be done, care must be taken to reverse the comparison head and also the lamps under test, unless complete reversibility of each has been shown to yield identical results.

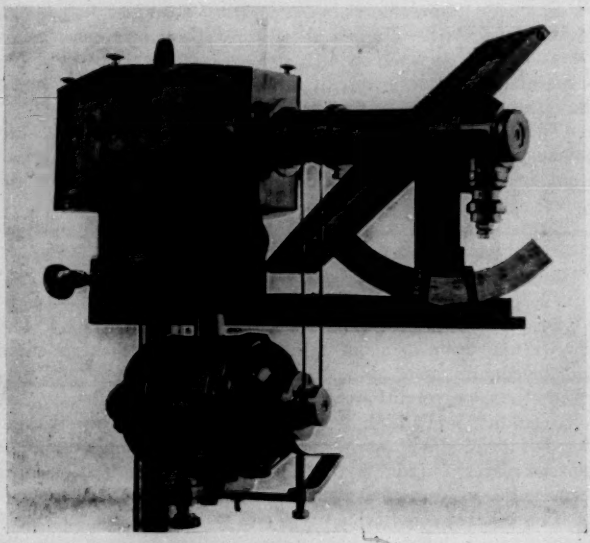
The method of securing a balance with the flicker photometer mentioned above should be emphasized here. The relative intensities of the two lights under test should be capable of being easily and quickly varied over a considerable range. One method of doing this is shown below.

In making a setting the light should be varied back and forth

over the region of minimum or no-flicker, and the limits gradually narrowed down to a balance. It is impossible to obtain good precision or accuracy by leaving the setting at any one point and trying to decide whether it is the correct point. There is no criterion for the judgment unless the intensity is changed.

The last general requirement is one of convenience. The parts

FIG. 1.



The Lummer-Brodhun with flicker attachment, motor, and neutral tint-absorbing screen.

should be easily accessible for cleaning, repairing, and demonstration.

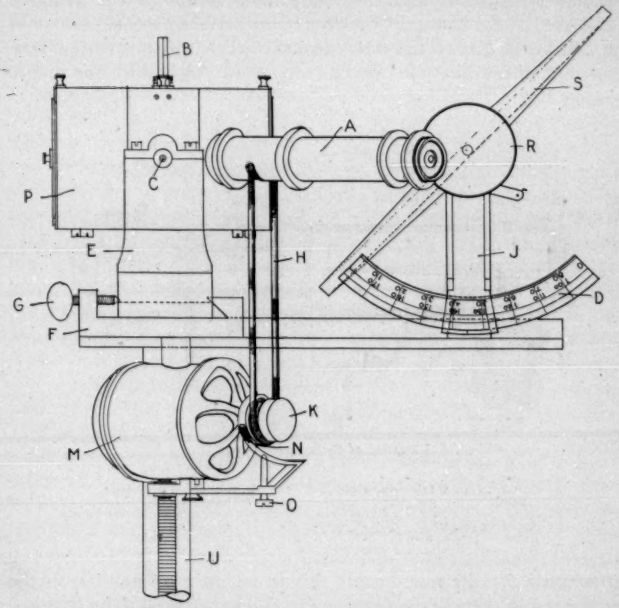
The attachment, with the motor and a neutral tint-absorbing screen, is shown in position on a Lummer-Brodhun head in Fig. 1.

Fig. 2 shows an elevation of the same.

The attachment (*A*) screws into the head in place of the regular telescope. The only other change necessary in the Lummer-Brodhun is the removal of the contrast glasses, which

the heads are built to permit readily. A belt (H) runs from the rotator within the tube to the motor (M), which is fastened to a clamp (F) that fastens to the horizontal casting beneath the box. The belt (H) is made of spirally-wound steel wire. It acts

FIG. 2.



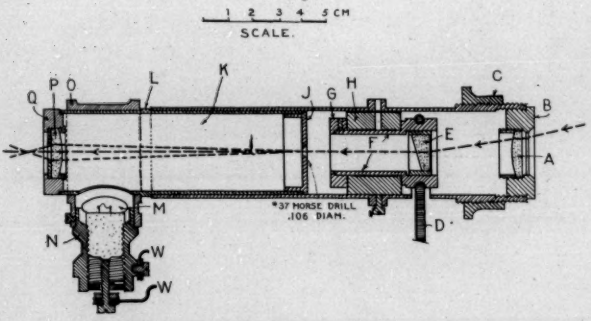
Elevation of Lummer-Brodhun with flicker attachment, motor, and neutral tint-absorbing screen, showing the method of clamping it to the head.

A. Flicker attachment.
B. Comparison disk.
C. Axis of head.
D. Scale.
E. Casting supporting head.
F. Clamp.
G. Clamp screws.
H. Belt.
J. Index.
K. Pulley.
M. Motor.
N. Friction brake.
O. Brake screw.
P. Lummer-Brodhun box.
R. Screen knob.
U. Support.

as a spring to take up any slack, and allows of the belt being easily slipped off the motor pulley. The part shown to the right is an addition, proposed by Dr. H. E. Ives, to facilitate reading, in the shape of a neutral tint-absorbing screen formed of parallel wires

in the frame (S). The frame is turned by knob (R) and the transmission read on the scale (D). This screen permits the light intensity to be easily and quickly varied over a considerable range, the desirability of which was mentioned above. In most cases it is more convenient to use such a screen than to move the light or the photometer head, though originally the attachment was intended to be used as the Lummer-Brodhun contrast head would otherwise be. Dr. Hugo Kruss[3] has a screen attached to his contrast photometer that would likewise undoubtedly work nicely with the flicker

FIG. 3.



Elevation of flicker attachment.

A. Front lens.
B. Collar for lens.
C. Collar to hold telescope rigid in head.
D. Belt.
E. Rotating prism.
F. Rotator bearing.
H. Stationary bearing for rotator.
J. Front of eye-piece.
K. Eye-piece.
L. Collar on eye-piece.
N. Lamp socket.
O. Rotating collar for lamp.
P. Eye-lens.
Q. Collar for lens.
W. Wires to lamp.

attachment, if the micrometer thread is not too fine. O on the motor is a friction brake designed to allow the speed to be changed quickly and easily over a wide range.

The attachment is shown internally in elevation in Fig. 3.

The right end of the tube screws into the Lummer-Brodhun, and when the tube is in position, with the belt (D) downward to the motor, the collar (C) is screwed tightly against the box. This brings the attachment central and square on the head and at the same time renders it rigid. A is a lens setting in a collar (B) that magnifies the Lummer-Brodhun field properly. E is a prism

[3] *Jnl. f. Gasbel.*, p. 457, 1914.

that sets in a cylindrical piece (G), the whole being rotated in bearing (H) by the belt (D). The bearing (H) is held in position by a screw beneath the tube. Upon loosening this screw the bearing with the rotating piece containing the prism can be slipped out of the tube either way if the belt (D) is first unhooked and pulled out. K is the eye-piece, which consists of a brass tube covered at the far end with a disk (J), in the centre of which is the hole making the 2° field mentioned above. The eye lens is at P, set in a collar (Q). The entire interior of this eye-piece (K) is painted white, and at M is placed a small 2-volt pocket torch-light which, when lighted, sends its rays up in the eye-piece. This, in turn, being a matt white, acts as an integrating sphere and diffuses the light so well that in looking into the eye-piece through the lens one sees surrounding the 2° opening a uniformly-illuminated surface. To allow the adjustment of the brightness of this surface to suit the observer, the socket (N) into which the light sets is fastened to a collar (O) which turns on the eye-piece (K) and moves the lamp over the solid part of the eye-piece, the whole arrangement acting as a shutter. It allows of the illumination being changed from too bright to absolute darkness and at the same time always keeping it uniform. A spring in the collar (O) pressing on the eye-piece insures the collar always turning smoothly. This collar can be slipped off the head of the eye-piece. The lamp socket (N) can be slipped out of the collar on loosening a set screw to allow the replacement of the lamp. L is a small collar fastened to the eye-piece that limits the distance which it can be slipped into the attachment tube, and at the same time the turning collar (O) bears against it on the opposite side. When it is desired to remove the eye-piece (K) it is simply pulled out of the other tube, bringing J with it. The white surface is thus never exposed to careless handling. Fixed resistances, not shown, go in series with the lamp and motor. They both are intended to operate on 110-volt line to the head.

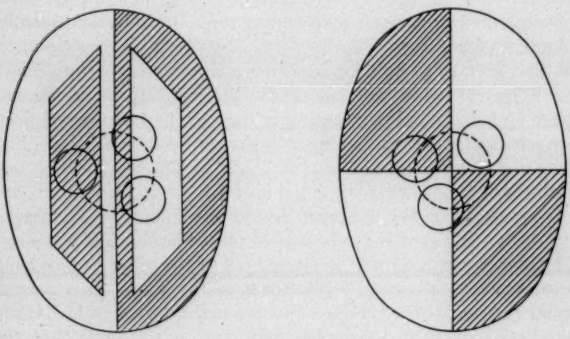
The method of cutting the Lummer-Brodhun field is shown in Fig. 4 A. The dotted line shows how it goes from trapezoid to trapezoid, the circles representing three different positions of the rotating prism. B shows the four-part field used in the laboratory flicker photometer recently built in this laboratory.[4] This four-part cube can be easily put in a Lummer-Brodhun box to

[4] *Phys. Review*, iv, p. 222, 1914.

replace the present one, if this type of field is preferred with the attachment. A question to be answered by the construction of this instrument was whether the irregular type of alternation given by the Lummer-Brodhun field would result in less precision or accuracy than the four-part field. No difference was found on trial, the irregular alternation of fields apparently calling merely for a lower speed.

The real value of such an instrument as described is best shown in actually measuring large color differences. A deep red monochromatic signal glass whose transmission was accurately known was measured against a 1.15 w.p.c. tungsten light on the

FIG. 4.



To show how the Lummer-Brodhun field (*A*) is cut by the attachment; also a four-part field (*B*).

other side; likewise a green signal glass and a 4 w.p.c. carbon lamp against the tungsten. In each case it has been found equal in precision and accuracy to the large laboratory instrument just referred to. The focus of the optical system of the attachment is in space and not on a surface, and, although the attachment was tested on a photometer head whose mirrors were badly corroded and needed replacing, no trouble was experienced from mechanical flicker.

It should be made plain that this attachment does not obviate the necessity of a proper selection of observers if the result is to represent that of an average eye. At least five observers should

be used in this case, as has been carefully explained in a paper recently published concerning this point, to which the reader is referred.[5] This is required of all flicker photometers.

In addition to the above point, care should be taken to remove the contrast glasses of the Lummer-Brodhun in using the attachment. If this is not done there will always be an outstanding intensity flicker. This point has been mentioned previously, but it is repeated here for emphasis.

The thanks of the author are due to Dr. H. E. Ives for many helpful suggestions, and especially to Mr. C. B. Smith, the laboratory's mechanic, who has taken a deep interest in perfecting many mechanical details.

PHYSICAL LABORATORY,
The United Gas Improvement Company,
May, 1915.

Electroplating with Cobalt. C. H. BUCHANAN and THOMAS HADDOW. (*The Metal Industry*, vol. 13, No. 6.)—This paper, prepared for the New York Branch of the Electroplaters' Society, contains data and results of experiments covering a long period, made by the writer, C. H. Buchanan, and a report of experiments of recent data in collaboration with Thomas Haddow. Early experience showed that a chloride-cobalt solution deposited more rapidly than did a nickel solution. Subsequent work done with cobalt proved so satisfactory that if the cobalt metal could have been obtained in sufficient quantities from commercial sources, even at a cost higher than paid for nickel, it would have been adopted for general work. The results of recent tests are summarized as follows: Cobalt plating has a beautiful bluish-white color. The deposit does not tarnish as readily as nickel; it is homogeneous, with a fine, close grain; it is smooth and not brittle, and will easily withstand bending tests. The time required in order to secure a satisfactory deposit is much less with cobalt than with nickel, and the current density with cobalt may be greater. Metallic cobalt costs more than nickel, but the cost of the salts is of small importance in a comparison of the two as to economy in results. Because of the greater conductivity of cobalt as compared with nickel, a current of higher density may be used in combination with a solution of less concentration. The time required in the solution is, with cobalt, one-third that required for nickel, and there is a similar saving of time in the buffing-room. The substitution of cobalt for nickel would thus greatly increase the speed of production in any established plant.

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[5] *Trans. Ill. Eng. Soc.*, 1915; *Phys. Review*, v, p. 230, March, 1915.

Expansion of Briquetting Industry. (*Commerce Reports*, 1915, No. 150.)—Although the number of briquetting plants in the United States was reduced from seventeen in 1913 to sixteen in 1914, there was a substantial increase in output of briquetting fuel. In this collateral branch of the coal-mining industry, which is still in the early stages of development, the smaller plants are going out of existence and the new establishments are of greater capacity. The briquettes favored in the Eastern States are egg-shaped and about the size of anthracite nut. In the central and Pacific coast States the popular type of briquetted fuel appears to be the larger size, about that of egg coal, for which the raw materials available seem best adapted. The material available for manufacturing briquettes in the Eastern States is culm from Pennsylvania anthracite mines and southwestern Virginia semi-anthracite mines and non-coking bituminous slack. Slack from the non-coking bituminous and sub-bituminous coal of the middle western and Rocky Mountain States is another cheap and abundant raw material for briquettes. The vast and almost untouched areas of lignite in North Dakota and Texas contain enormous supplies of fuel that European practice has taught is well adapted to briquetting, and is much more usable in that form than in a raw state.

Talc and Soapstone. (*U. S. Geological Survey.*)—The United States produces more talc and soapstone than all the rest of the world combined. Moreover, according to J. S. Diller, of the United States Geological Survey, our production has nearly doubled in the last decade, increasing from 91,185 short tons, valued at $940,731, in 1904 to 172,296 short tons, valued at $1,865,087, in 1914. Of talc alone the United States produced 151,088 tons, and of soapstone 21,208 tons.

Talc is a simple mineral of which soapstone is an impure massive form. Few people are aware how much we owe to talc and soapstone. It is one of the softest of minerals. It is so smooth and slippery that it has become a great panacea for friction in many branches of human industry.

There are nine States producing this useful mineral. New York continues to be the leading producer, yielding more than 57 per cent. of the total production of talc in the United States and far outranking all other States excepting Vermont, which has in recent years so greatly increased its production that in 1914 its output was about three-fourths that of New York. Of soapstone, Virginia holds the greatest supply, and backed up by Vermont it meets the great demand for washtubs, sinks, and fireless cookers.

Talc is used in making talcum toilet powder, the tailor uses it to chalk fabrics for new suits, and talc "slate pencils" and crayons have enabled many scholars to solve knotty problems. Talc bleaches our cotton cloth, and in paints we see it everywhere, but its chief use is as a filler in paper of many kinds, from roofing to news paper.

NOTES FROM THE U. S. BUREAU OF STANDARDS.*

THE USE OF SODIUM SALTS IN THE PURIFICATION OF CLAYS AND IN THE CASTING PROCESS.

By A. V. Bleininger,
Ceramic Chemist, Bureau of Standards.

[ABSTRACT.]

THE present work deals with a technical investigation of the effect of small quantities of sodium carbonate and sodium silicate in the separation of the impurities from kaolins and in reducing the water content in raw porcelain mixtures composed of kaolin, ball clay, feldspar, and quartz, ground together in water to form a suspension which is used to cast clay wares.

It has been shown that kaolinitic clays may be decidedly improved by carrying on the washing process with water containing a definite, small amount of caustic soda or a mixture of caustic soda and sodium silicate. This makes possible more complete sedimentation of the granular impurities which it is desired to remove. This treatment improves the color of most clays, enriches their content in clay substance, decreases the drying shrinkage, but tends to increase the burning shrinkage. Clays containing iron in the form of finely-divided ferric oxide cannot be treated successfully by this process.

The electrical osmosis process, invented by Count Schwerin, consisting in collecting the negatively-charged clay and quartz particles on the positive electrode under the influence of a direct current, plays no important part in the purification of clays. Its function seems to be purely mechanical. It thus replaces the filter press, and if the latter is of the modern improved type any advantage resulting from continuous and automatic operation will be more or less neutralized. The higher the voltage of the current employed the denser will be the deposit of clay and the lower in water. The preliminary sedimentation process using caustic soda or other electrolytes seems to be the real feature of the osmose process. The electrical method of separation cannot remove finely-

* Communicated by the Bureau.

divided ferric oxide. It also deposits with the clay any finely-divided quartz which may be in suspension.

The efflux viscosimeter appears to be satisfactory for the technical study of casting slips.

Sodium silicate is a more effective reagent, so far as the reduction of viscosity is concerned, than sodium carbonate.

North Carolina kaolin in suspension is not influenced by the reagents in question in a marked manner, due to its comparative coarseness.

Georgia kaolin has a far greater initial viscosity than the North Carolina kaolin, and appears to be very sensitive to additions of sodium carbonate. It shows a well-defined minimum of short range, which rapidly turns toward a maximum. Its range at minimum viscosity is greater when sodium silicate is used.

Tennessee ball clay No. 3 gives no indication of a decided minimum viscosity point.

Florida kaolin responds decidedly to the addition of sodium carbonate, but a well-defined minimum is not reached, probably due to the comparatively large amount of pseudo emulsoid or lyophile substance produced. With sodium silicate the viscosity decrease is equally great, with no tendency to approach another maximum value.

A large volume of air seems to be absorbed in clay slips, the removal of which increases the viscosity. Thorough and vigorous stirring of the slip, therefore, seems essential, and exhaustion by vacuum might be beneficial. The effect of the entrapped air was more prominent with lower alkali concentrations and with the Georgia and Florida kaolins.

A body containing North Carolina kaolin, Florida kaolin, and Tennessee ball clay proved to respond well to the addition of sodium carbonate, but showed a turn towards greater viscosity (recoagulation) for additions greater than 0.06 per cent.

A commercial slip, fairly high in English ball clay, showed a rapid drop in viscosity, but required a much higher per cent. of the reagent mixture than the single clay bodies. The viscosity seemed to remain constant at the minimum value. It is possible that this condition is due to the presence of organic colloids which exercise a protective influence.

Interaction occurs between different clays with respect to viscosity changes, independent of the alkali additions, and these are

not necessarily proportional to the ratios of the admixed clays or regular in character. It is important, therefore, to know just what this interaction is, since it is quite possible that some clays may have a detrimental effect upon the others in making up the casting body. The ball clay content in a casting slip should be reduced to the minimum.

Absorption of the alkaline reagents plays an important rôle in casting slips. Sodium hydroxide is absorbed to a far greater extent than the carbonate, and probably combines chemically with the clay.

The tensile strength of cast bodies containing alkali additions may be greater than that of the same materials cast without these reagents. Single clay body slips, used in the state of maximum deflocculation, do not seem to be weaker than when the suspensions are carried only to a point to the left of minimum viscosity.

Casting bodies with low concentrations of alkali are more apt to change their viscosity with time than those containing larger additions.

AN INVESTIGATION OF FUSIBLE TIN BOILER PLUGS.

By G. K. Burgess, Physicist, and P. D. Merica, Assistant Physicist.

THE failure and deterioration of fusible tin boiler plugs in service have been investigated. In some cases such plugs have failed to melt and so give warning of dangerous boiler conditions, and investigation has shown that the tin filling in these cases had become oxidized to tin oxide (SnO_2), which has a melting-point above 1600° C. (2900° F.).

About 1050 plugs, of which 100 were plugs which had been in service, were obtained through the courtesy of the Steamboat Inspection Service, Department of Commerce, and subjected to examination. This included inspection of design and construction, condition and purity of the tin filling, and, in the case of the used plugs, their classification according to the type of deterioration undergone by them in service.

One pronounced and dangerous type of deterioration is the oxidation of the tin along the grain boundaries by which is formed a network of oxide throughout the tin. The plugs showing deterioration of this type all came from the same manufacturer and contained zinc in amounts varying from 0.3 to 4.0 per cent. It

is shown that this type of oxidation is due to the presence of this zinc. The latter metal is not soluble in the solid state in tin, and when a tin with small amounts of zinc is heated as in a boiler to about 180° C. (340° F.) this zinc coalesces as a network enveloping the tin crystals or grains. The boiler water, particularly if it contains alkali, will attack the zinc, eating its way into the alloy along the zinc network, and finally form the oxide network described.

Lead and zinc are found to be the principal impurities in tin plug fillings, and, since all " failed " plugs contained these or other impurities, the conclusion is reached that if these impurities are eliminated by strict specifications and inspection, which will allow only admittedly superior qualities of tin, such as Banca and some others, the danger of failures of these plugs will no longer exist.

In order to test quickly the purity of a tin sample a determination of its freezing-point may be made. This can be done conveniently with two or three grammes of the sample, and requires only inexpensive apparatus, including a high-resistance millivoltmeter and a copper-constantan thermocouple.

ELECTROLYSIS MITIGATION.*

THE paper gives a general discussion of the subject of electrolysis, comprising a comparison of electrolysis conditions in America with those in other countries, a discussion of electrolytic corrosion proper as distinguished from self-corrosion, and touches briefly on electrolysis effects in concrete and steel buildings. There is also a brief discussion of the effects of stray currents other than corrosion, such, for example, as the production of fires or explosions.

Methods of electrolysis mitigation are discussed at length. All of the various methods of mitigation that have been proposed or tried are discussed under two main heads: First, those methods applicable to underground pipe and cable systems; and, second, those applicable to the railway negative return. Those methods applicable to underground pipe systems comprise the following: (1) Surface insulation of the pipes; (2) chemical protection,—that is, rendering the pipe surface passive by surrounding it with earth filled with lime or other chemical that will prevent corrosion;

* Abstract of Technology Paper.

(3) cement coatings; (4) cathodic protection,—that is, maintaining the pipe or cable always negative to earth by means of a motor generator set or battery or other sources of electromotive force; (5) favorable location of pipe with respect to tracks; (6) the use of non-corrodible conducting coatings; (7) electric screens; (8) the use of insulating joints in pipes; and (9) pipe drainage.

The discussion leads to the conclusion that of the various methods under this class that have been tried none are suitable for general use as primary means of preventing electrolysis trouble. The methods of chemical protection, cement coatings, cathodic protection, and conducting coatings should be regarded as substantially worthless in their present state of development. Surface insulation of pipes by means of paints or dips is not much more reliable, but insulation by putting pipes in troughs or conduits filled with pitch may be used in special cases where the expense would be justified. The practice of placing all pipes as far as possible from railway tracks affords a certain measure of protection, of which advantage should always be taken wherever practicable in laying new lines or relaying old ones. The use of electric screens is often a valuable expedient in taking care of acute local cases of trouble in existing mains.

These methods, with the exception of that relating to the proper location of pipes in new work, are suitable only to special conditions, however, and are not usually to be considered as important factors in any general plan for electrolysis mitigation.

Pipe drainage is sometimes useful, but should be used with proper restriction and with due precautions against setting up any dangerous conditions either in the system drained or in neighboring systems. In general, in city networks where there are a number of independent underground systems to be protected pipe drainage should be used as little as possible, the chief reliance being placed on mitigative measures applied to the railway negative return. The drainage of lead cable systems will, however, usually be desirable, and these should always be drained by means of suitable insulated feeder systems so arranged as to drain the least practicable current from the cables in order that neighboring structures may not be subjected to unnecessary danger thereby.

The most valuable mitigative measure that can be applied to the pipe system consists in the proper use of insulating joints, and the extensive use of such joints should be encouraged in new

work and in making repairs. Precautions are necessary in their use, however, and these are set forth in the discussion.

Taking up the methods applicable to the railway system, there is a very brief reference to the alternating-current system, double-trolley system, the use of negative trolley, and the periodic reversal of trolley polarity. This is followed by a somewhat extended discussion of methods of reducing potential differences in the uninsulated portion of the negative return. These various means comprise: (1) Proper construction and maintenance of way; (2) grounding of tracks and negative bus; (3) use of uninsulated negative feeders; (4) use of insulated negative feeders without boosters; (5) use of insulated negative feeders with boosters; (6) three-wire systems; and (7) proper number and location of power-houses.

The discussion leads to the conclusion that the alternating-current system, the double-trolley system, the use of negative trolley, the periodic reversal of trolley polarity, and the use of uninsulated negative feeders in parallel with the rails, when considered solely as methods of electrolysis mitigation, are either impracticable or else open to the objection that the expense or operating difficulties attending their application are rendered unnecessary because of the fact that there are other adequate methods available for general application which are comparatively cheap to install and which introduce but slight complications into the operating system.

The importance of proper construction and maintenance of track return is emphasized, and the drainage of the roadbed where practicable is urged. Also where the track is laid on private right of way the rails and ties should be kept as far as possible out of direct contact with the earth by the use of good rock ballast.

It is also pointed out that the three-wire system, when viewed solely from the standpoint of electrolysis mitigation, possesses large possibilities. Attention is called to the fact, however, that up to the present time sufficient experience has not been had with this system to determine whether it is practical from the operating standpoint under average conditions of service. It is therefore urged that experiments with this system be made under conditions to which it is best adapted.

The most effective methods that have been thoroughly tried out in practice over long periods are the use of insulated negative

feeders either with or without boosters, generally the latter. In most cases where the feeding distances are not too long an insulated feeder system without boosters will prove cheapest and at the same time more satisfactory because of its greater simplicity. It is possible, by the proper application of such systems, to reduce the potential gradients in the earth to such low values that in most cases little damage would result. In many cases, however, it may be better, where conditions are favorable, to combine one of these methods with either the insertion of a moderate number of insulating joints in the pipes or with the use of a very limited amount of pipe drainage, provided local conditions are favorable to the use of this method. The insulated feeder system would be applied to reduce the potential gradients throughout the system to very low values, and one or the other of the auxiliary systems used to eliminate largely any residual electrolysis that might still remain.

The last section of the paper is devoted to a discussion of regulations regarding electrolysis mitigation. The subject of what criteria should be used for determining the adequacy of electrolysis conditions is taken up at some length, and it is shown that potential measurements showing a total drop of potential in the railway negative return and also potential gradient measurements throughout the track network are very valuable. It is also shown that all-day average values of these potential readings give a better criterion of the actual danger from electrolysis than any short-time peak value. It is recommended that in fixing voltage limitations some plan analogous to the zone system should be adopted, the voltage limits prescribed for the various zones being determined largely by the degree of development of the underground utilities in the various zones. The voltage drops either in the tracks or in the pipes, and earth may be used as the basis for fixing limitations, but in general the latter is to be preferred. The question of what constitutes a safe limit for voltage drops in the track return is discussed at some length, and the conclusion is reached that for the overall potential drop in the railway tracks a limit of from two to four volts is reasonable and adequate, and the potential gradient should in general be restricted to 0.3 or 0.4 per thousand feet, these figures being all-day average values. Where short-time peak values are used, the figures would, of course, be considerably higher.

In order that ready determination of voltage drops may be made at any time, potential wires should be installed running from some central point to selected points on the railway or pipe networks. These points should include the points of approximately highest and lowest potential and preferably, also, some intermediate points. It is recommended that exemption from any regulations regarding track voltages should be made in special cases as described in the paper where local conditions make it improbable that any serious damage would result. The responsibilities of the owners of underground utilities regarding the mitigation of electrolysis troubles is discussed, and it is recommended that any regulations governing electrolysis mitigation should be made to apply not alone to the railway system, but should also define the responsibilities of the owners of underground utilities, since the latter can often contribute materially to the diminution of the trouble at a practically negligible cost.

THE VISCOSITY OF PORCELAIN BODIES HIGH IN FELDSPAR.

By A. V. Bleininger and C. S. Kinnison.

IN this work the degree of softening of previously-fired porcelain bars subjected to a tensile stress of 14.5 pounds per square inch, at a series of temperatures between 1275° and 1380° C., was determined by measuring the elongation. The object of these tests was to obtain data for selecting compositions showing the greatest rigidity at the highest kiln temperatures for the purpose of selecting bodies causing least loss in manufacture due to deformed ware.

Higher contents of feldspar increase the viscosity of low clay compositions markedly. In a 45 per cent. kaolin-clay body, 35 and 40 per cent. of feldspar are most effective in decreasing the viscosity, but higher percentages again increase it. Still higher clay contents cause the softening effect of feldspar to diminish decidedly. Flint, in high feldspar bodies, does not seem to be an important factor in governing the viscosity. For high temperature porcelains high clay content is essential. High clay bodies having feldspar as the only other constituent show very great viscosity.

The function of viscosity is not a continuous one. After attaining a certain degree of softening, a well-defined rigid state

ensues, which does not tend to approach further softening until a temperature of 1345° has been reached. Beyond this point the second stage of decided softening is entered upon. Although showing the same degree of viscosity at 1388°, bodies may be divided into several classes, according to the degree of softening observed between 1300° and 1345°. Within this temperature range greatest rigidity was shown by the following compositions:

Kaolin and clay, per cent.	Feldspar, per cent.	Flint, per cent.
40	30	30
45	35	20
45	50	5
50	30	20
50	50	..

STANDARDIZATION OF BOMB CALORIMETERS.*

There has just been issued by the Bureau a paper describing briefly the methods of calibrating and using bomb calorimeters, such as are used in determining the amount of heat available from a given weight of coal or coke or other combustible. The amount of heat which can be obtained depends largely upon the kind and quality of fuel. When purchased in large quantities a fuel is commonly tested to determine the amount of heat available per pound, and the price paid depends upon the results of these tests.

The instrument used for such tests is called the bomb calorimeter and consists essentially of a steel shell or "bomb" in which a small weighed sample of the fuel can be burned in pure oxygen gas. The bomb is immersed in a known amount of water before the sample is ignited, the heat produced warms the water, and by suitable measurements of the change of temperature the amount of heat can be calculated.

Provision is made by the Bureau of Standards for standardizing bomb calorimeters by means of standard samples of certain pure materials, viz., sugar, naphthalene, and benzoic acid. By burning known amounts of these substances in the bomb the observer determines the amount of heat required to raise the temperature of the bomb, together with the proper amount of water, one degree. This being determined, the amount of heat furnished by a given sample of coal burned in the same bomb with the same amount of water can be found.

* Circular No. 11.

Thus these standard samples, which are sent all over the United States, serve as standards of heat and make it possible to get the same results from tests made anywhere in the country, much as the use of the standards of length and of mass makes a yard or a pound the same in all parts of the country.

THE TESTING OF RUBBER GOODS.

THE Bureau is about to issue the third edition of a circular on the testing of rubber goods This publication, which has been very much enlarged, is fully illustrated and describes in detail the method of procedure in conducting physical and chemical tests of rubber. The testing machines and apparatus developed at the Bureau of Standards greatly facilitate the testing of rubber, and the object of this circular is to assist manufacturers and consumers in establishing standard specifications and standard methods of test. The subject matter proper is introduced by a brief outline of the processes through which rubber passes before reaching the factory, followed by a short description of the usual processes of manufacture, which include washing, drying, compounding, "making up" various articles, and vulcanizing. The physical tests most commonly employed are explained very thoroughly. These include tests for tensile strength, ultimate elongation, and elasticity. Conditions affecting the results of tests are discussed at some length, and experimental data are given to show the necessity of a standard procedure in testing.

A general discussion of the chemistry of rubber is followed by a brief explanation of the object of each of the analytical determinations that are commonly made. After this there are given in detail the methods in use at the Bureau for each of these determinations. They are not entirely original, but have been compiled from the various publications on rubber analysis, from the information gained through the routine testing of rubber goods for delivery on government contracts, and from coöperative research with various scientific organizations.

The tentative method of analysis and specification for 30 per cent. Hevea insulation compounds, adopted by the joint Rubber Insulation Committee, is next given.

A bibliography listing the more important books and journals devoted to rubber, and the Bureau's regulations regarding the testing of rubber goods conclude the circular.

NOTES FROM NELA RESEARCH LABORATORY.*

THE BRIGHTNESS-DIFFERENCE SENSIBILITY OF THE EYE UNDER VARIOUS BRIGHTNESS OF TEST-FIELDS AND SURROUNDINGS.†

By Percy W. Cobb.

THE problem consists in preparing a bright surface which shall fill the entire visual field and form the surroundings of a small field of independently variable brightness. The former is obtained by a device somewhat on the principle of the Ulbricht sphere, made in the form of a cube of one metre side, with its interior edges and corners cut off to form a polyhedron approximating a sphere. This is provided with an opening in one side, in which the observer's face is inserted, and the interior is illuminated through a diffusing glass in the oblique surface immediately above this. A small rectangular opening opposite the observer, 3.4×4.6 cm. and one metre away, forms the outline of the test-field, and is the plane of focus of an image formed by a projecting lens of a small illuminated milk-glass. A condensing lens just beyond makes the image appear to the observer to fill the opening. A second image formed by partial reflection and projected by the same lens is made to fall in the same place, and a device arranged so that it may be made to cover the right or the left half only of the test-field, as brightness added to that half, varied by means of an adjustable sector disk and measurable independently of the field-brightness itself.

Fourteen sets of conditions were used, namely: one in which field and surroundings were equal at 5.5 millilamberts (17.5 candles per square metre), seven (a to g) in which the field alone was varied from these conditions from 137 to 0, and six (u to z) in which the surroundings alone were varied from 38 to 0 millilamberts (436 to 0 and 121 to 0 candles per square metre). The results given in the table are each the mean results of 500 judg-

* Communicated by the Laboratory.

† To appear in *Psychological Review.*

ments of each of three observers. They are stated (1) as the absolute value of the addition to the half of the field necessary to be just visible, and (2) as the ratio that this bears to the field before addition.

TABLE I.

Absolute and Relative Difference-threshold for Various Conditions.

	Brightness conditions Millilamberts *			Difference-threshold Millilamberts	
	Field	Surroundings	Ratio	Absolute	Fractional
a.	137.1	5.43	25.2	0.918	0.0067
b.	40.1	5.43	7.39	0.2373	0.0059
c.	11.09	5.43	2.04	0.0647	0.0058
o.	5.52	5.43	1.02	0.0289	0.0053
d.	2.803	5.43	0.52	0.0207	0.0074
e.	0.66	5.43	0.12	0.00952	0.0143
f.	0.1696	5.43	0.031	0.00672	0.0396
g.	0.000	5.43	0.000	0.00616	
u.	5.49	38.15	0.14	0.0672	0.0123
v.	5.52	14.44	0.38	0.0396	0.0072
o.	5.52	5.43	1.02	0.0289	0.0053
w.	5.52	2.715	2.03	0.0327	0.0059
x.	5.52	0.674	8.19	0.0371	0.0067
y.	5.52	0.1719	32.1	0.0421	0.0076
z.	5.52	0.000		0.0456	0.0082

* A lambert is defined as the brightness of a perfectly diffusing surface emitting one lumen per square centimetre, equal to a brightness of 0.3183 candle per square centimetre. A millilambert is 0.001 lambert.

The fourth column gives the ratio of brightness of field to brightness of surroundings. The sixth column the ratio of the threshold amount of brightness added to one-half of the field (column 5) to the original brightness of the field (column 2).

It is to be noted that with constant value of surroundings the absolute value of the difference rises as the field-brightness is increased (a to g). On the other hand, where the field is constant and its surroundings variable, the difference is minimal at the point where the latter equals the field-brightness, otherwise greater whether the surroundings exceed or fall below the field in brightness. These relations are more clear when we think of the threshold-difference as a fraction of the field-brightness (column 6) and compare it with the ratio of field-brightness to surroundings-

brightness (column 4). In this case the former rises rapidly (in both series) as the latter decreases, and rises also as the latter increases, but only moderately.

ENERGY-DENSITY IN THE EYE-MEDIA.*

By M. Luckiesh.

IN a previous paper on "Radiant Energy and the Eye" † the results of a large number of computations were presented to show the relative amounts of energy absorbed in various parts of the eye-media when beams of radiation from various illuminants pass through the eye as in ordinary vision. This work has been continued in a study of the mean energy-density in the various vertical sections of the eye under conditions of illumination from sources emitting (or reflecting) radiation of various spectral characters. Special attention is called to the widely different conditions in the eye; namely, when viewing objects subtending at the eye small and large solid angles, respectively. When the viewed object subtends a small angle, the mean energy-density throughout the eye remains approximately constant until the retina is approached. In other words, the concentrating action due to refraction is overcome by the progressive absorption of the eye-media. At the retina the mean energy-density becomes relatively very great. In the case of objects subtending a large solid angle the mean energy-density is much greater in the lens than at the retina. The question is important, in view of the fact that cataract and eye-fatigue are considered by many to be due to absorption of energy. As a practical application of the data obtained by computation, it is seen that a molten glass surface, such as a large pot of molten glass, appears as harmless as a small surface at the same distance (the retinal images being of the same brightness); however, the energy-density in the lens will be many times greater in the case of the surface subtending the larger solid angle at the eye. In fact, the ratio of the energy-densities in the lens (in the pupillary plane) will be approximately the same as the ratio of the solid angles subtended by the large and small surfaces, respectively.

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* To appear in the *Electrical World*.

† *Electrical World*, October 25, 1913.

VISUAL PATTERN-DISCRIMINATION IN THE VERTEBRATES: III.—DIFFERENCE-THRESHOLD FOR BAND-WIDTH IN THE MONKEY AND THE DOMESTIC CHICK.*

By H. M. Johnson.

IN the experiments reported in the above series of papers, the general procedure observed is that of the so-called "discrimination-method" of Professor R. M. Yerkes. In brief, the animal is placed in a chamber in which his movements may be controlled and observed, and allowed to obtain food in one of two food-compartments. The entrance to each compartment is marked with a visual object. These objects are mutually interchangeable, and may be made to differ in any physical characteristic—*e.g.*, area, outline, brightness, color, etc. One of these objects, the "positive stimulus," indicates the food-compartment to be chosen; the other, called the "negative stimulus," indicates the food-compartment to be rejected. If the animal chooses the box indicated by the negative stimulus he receives a light electric shock, automatically given. The left-right position of the stimuli is determined by a pack of shuffled cards containing an equal number of reds and blacks. Numerous well-known precautions described in Papers 1 and 2 of this series † are taken to prevent other factors than the visual stimulus difference under study from becoming effective.

In the beginning of the work on a given problem the stimulus-difference is made sufficiently large for the animal to acquire a "perfect discrimination-habit"—*i.e.*, of choosing the positive stimulus in 90 per cent. to 100 per cent. of a large number of trials. The difference is then decreased by very small steps until a point is found where the percentage of correct choices is about 75—*i.e.*, where the stimulus-difference is effective in half the total number of cases. This point is called the animal's threshold.

In the present work the stimuli are prepared by two Ives-Cobb visual acuity test-objects, so constructed as to present two fields, respectively, equal in area, outline, color, and mean brightness. Each field is covered by a system of alternate dark and bright striæ, respectively, equal in width, and the width and direction of the striæ may be independently varied.

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* To appear in *Journal of Animal Behavior.*

† *Journal of Animal Behavior*, vol. 4, p. 319 ff., 1914.

In a preliminary experiment the direction of both systems was made horizontal, and the width of each band on the negative field was made about 0.1 mm. (The animal was restrained from bringing the eye nearer than 60 cm. from the test-object, at which distance the negative field was of sensibly uniform brightness to the human eye.) Each of the bands on the positive field was made 2.23 mm. wide until a perfect discrimination-habit was established, after which band-width was reduced by small steps until the threshold was found for each animal. For the one Capuchin monkey used in this work this value was 0.163 mm., each band subtending a visual angle of 57″. (The mean value obtained for five photometrists was 49″, with an individual mean value of 2 per cent. to 4 per cent.) For two Indian game cocks the band-width was a little over 0.7 mm., subtending a visual angle of a little over 4′ at the distance given. Two dogs failed to establish a discrimination-habit, although the band-width on the positive field was about 4 mm. These results served to determine the range of stimulus-values in which the present tests could be made.

In the work herein reported both systems of striæ lay in the horizontal direction, and the band-width on each field was above the animal's stimulus threshold; stimulus-difference under study was the band-width and conversely of number in the positive and negative systems, respectively. The animals were fed at the coarser system and shocked at the finer system.

Chick 1 failed to learn the problem. He succeeded, however, in maintaining perfect discrimination when the two systems were opposite in direction and also different in band-width, but he failed when either factor was eliminated.

Chick 2 was tested at six points between absolute band-width of 3.1 mm. and 0.74 mm. The thresholds yielded varied between 33 per cent. and 42 per cent. of the value of the "standard" stimulus. No consistent rule of variation was discovered.

The monkey was tested at seven points for the "upper" and "lower" threshold values, between 1.561 mm. and 0.173 mm. His results are quite consistent. At the standard stimulus-value 0.312 mm. the threshold was less than 3 per cent.; at 0.223 mm., about 5 per cent., and at 0.191 mm., 9 per cent. At stimulus-values larger than 0.312 mm. the thresholds were much larger, but they were obtained in the early stages of experimentation, and a later test revealed that the differences were due to a practice-effect.

A subsequent test gave a mean of the upper and lower thresholds at standard stimulus—0.780 mm. of less than 3 per cent.

The final values given by the monkey, when plotted against stimulus-values, gives curves which closely approximate those yielded by two human subjects, in rather rough tests, made under identical visual conditions.

A discussion of these results in the light of comparative anatomy and physiology will be attempted in a later paper.

Polluted Farm Waters. (*U. S. Geological Survey,* Water Supply Paper 255.)—In selecting a country home for the summer special attention should be given to the question of a pure water supply. Farms, which are generally remote from towns, cities, or other areas of congested population, seem to be almost ideally situated for obtaining pure and wholesome water. In reality, however, polluted water is exceedingly common on farms, and typhoid-fever rates are usually greater in country districts than in cities. Typhoid fever is transmitted largely in drink or food taken into the stomach, and is especially likely to be communicated by polluted waters obtained from shallow wells near spots where the discharges of typhoid patients have been deposited and subsequently carried into the wells, and it is doubtless principally this fact that makes the disease so common in farming regions.

The sources of the water used on the farm are numerous—lakes, streams, springs, wells of various kinds, and cisterns—all being extensively used, although the water from lakes and streams is generally used only for stock. Each of these sources under some conditions may yield entirely safe and satisfactory supplies, while under other conditions certain of them may be a constant menace to health.

Of the various sources of water supply, the ground water is the most satisfactory for farm use, because it is least liable to pollution, and streams and pools are the most unsatisfactory, because of the ease and frequency with which they are contaminated. Fortunately, however, they are very seldom used for drinking and domestic purposes, being utilized mainly for stock, on which the effect of slight general pollution is not serious, though some severe epidemics of disease among cattle have been traced to polluted streams. The underground supplies, whether from wells or springs, although safe in many localities, are far from being universally so, the safety depending mainly on their location and on the nature of their protection.

Many of the failures to protect adequately the water supplies used for drinking arise from a lack of knowledge of the manner in which waters circulate through the ground and of the ways in which they may become polluted.

THE FRANKLIN INSTITUTE

RESOLUTIONS ADOPTED BY THE BOARD OF MANAGERS ON THE OCCASION OF THE DEATH OF JOHN BIRKINBINE.

PHILADELPHIA, July 1, 1915.

The Board of Managers of The Franklin Institute, deploring the passing away of its honored member, the former president of the Institute, JOHN BIRKINBINE, unanimously resolves to place on record in the minutes of the Board, as an expression of regard and esteem by his associates, this present memento of the manifold services rendered by John Birkinbine to The Franklin Institute during the fifty years of his active participation in its work.

The Board further resolves that this memorial be published in the JOURNAL of the Institute and that a copy thereof be engrossed and transmitted to the family of its departed member.

RESOLUTIONS ADOPTED BY THE COMMITTEE ON SCIENCE AND THE ARTS ON THE OCCASION OF THE DEATH OF RICHARD GILPIN.

PHILADELPHIA, July 1, 1915.

The Committee on Science and the Arts of The Franklin Institute, sensible of the loss which its membership has sustained through the untimely death of RICHARD GILPIN, on June 21st, unanimously resolves that there be recorded in its minutes this present memorial of the faithful service rendered by Richard Gilpin as member of this body during the past eleven years, in recognition of his efficient collaboration as a technologist, and in appreciation of the sterling qualities of his genial personality.

The committee further resolves that this minute be published in the JOURNAL of the Institute and that an engrossed copy thereof be transmitted to the family of the deceased.

MEMBERSHIP NOTES.

NECROLOGY.

George Ormrod was born in Preston, England, on July 13, 1839, and died at Allentown, Penna., on June 21, 1915.

He came to the United States when nineteen years of age, and on the death of his uncle, William Donaldson, became superintendent of the latter's colliery at Tamaqua, Pa. Later he was associated with his father-in-law in the operation of a colliery, and from 1865 to 1877 operated, with several others, the Girard Colliery at Raven Run. Mr. Ormrod was also connected with St. Nicholas Coal Company, Mahanoy City, as director and president, and the Kehley Run mine at Shenandoah.

In 1880 he leased the Emaus Blast Furnace for the purpose of manufacturing pig iron, and three years later established the Emaus Pipe Foundry for the manufacture of cast-iron pipe. The latter firm was incorporated in 1886 as the Donaldson Iron Company and has been in continuous operation for over thirty-five years, Mr. Ormrod serving as manager and treasurer and later as president.

With his son-in-law, Mr. Ormrod organized the first Portland cement plant in New Jersey in 1893, and in 1897, with several others, organized the Lehigh Portland Cement Company.

Mr. Ormrod was active in numerous charitable organizations, clubs, and societies.

He became a member of The Franklin Institute in 1905.

Richard Gilpin, 904 Clinton Street, Philadelphia, Penna.

LIBRARY NOTES.

PURCHASES.

AGRICOLA, GEORGIUS.—De re metallica. Translated from the first Latin edition of 1556 by H. C. and L. H. Hoover. 1912.

BAUSCHINGER, J., and PETERS, F., ed.—Logarithmic-trigonometrical Tables. 2 volumes. 1911.

BROWN, JAMES CAMPBELL.—Essays and Addresses. 1914.

DALL, WILLIAM H.—Spencer Fullerton Baird—A Biography. 1915.

DAVEY, NORMAN.—The Gas Turbine. 1914.

DUNBAR, SEYMOUR.—History of Travel in America. 4 volumes. 1915.

FRIEND, J. N., and others.—Text-book of Inorganic Chemistry, volume i, parts 1 and 2. 1914.

GASTER, LEON, and DOW, J. S.—Modern Illuminants and Illuminating Engineering. 1915.

GIOLITTI, FEDERICO.—Cementation of Iron and Steel. 1915.

HAYES, HAMMOND V.—Public Utilities, their Fair Present Value and Return. 1915.

HERO.—Di Herone Alessandrino de gli Automati onero Machine se Moventi, Libridue, Tradotti dal Greco da Bernardino Baldi Abbate di Guastralla. Venice, 1589.

HUBBARD, C. L.—Combined Power and Heating Plants. 1915.

JEANS, J. H.—Mathematical Theory of Electricity and Magnetism. 1911.

JERVIS-SMITH, F. J.—Dynamometers. 1915.

London Chemical Society.—Annual Report, volume xi. 1914.

MILLS, A. P.—Materials of Construction. 1915.

NOEBELS, SCHLUCKEBIER, and JENTSCH.—Telegraphie and Telephonie. 1901.

PEARSON, KARL, ed.—Tables for Statisticians and Biometricians. 1914.

PEEK, F. W. J.—Dielectric Phenomena in High-voltage Engineering. 1915.

RAMELLI.—Le Diverse et Artificiose Machine del Capitano Agostino Ramelli dal ponte della tresia Ingegnerio del Christianissimo Re di Francia et di pollonia. Nellequali si contengono narij et industriosi Monimenti, degni digrandissima Speculatione per cauarne beneficio infinito in ogni forte d'operatione; Composte in lingua Italiana et Francese. Paris, 1588.

SCHOTT.—P. Gasparis Schotti, Technica Curiosa sive Mirabilia Artis Libris xii comprehensa quibus varia Experimenta, variaque Technasmata Pneumatica, Hydraulica, Hydrotechnica, Mechanica, Graphica, Cyclometrica, Chronometrica, Automatica, Cabalistica. Nuremberg, 1664.

TARTALEA.—Quesiti et Inventioni diverse de Nicolo Tartalea Brisciano. Venice, 1543.

WILLIAMS, H. E.—Chemistry of Cyanogen Compounds. 1915.

GIFTS.

American Institute of Mining Engineers, Transactions, vol. xlix, 1914. New York, 1915. (From the Institute.)

Amherst College, Obituary Record. Amherst, Mass., 1915. (From the College.)

Baylor University, Catalogue. Waco, Texas, 1915. (From the University.)

Bourne-Fuller Company, Tool Steel and its Uses. Cleveland, no date. (From the Company.)

Brown and Sharpe Manufacturing Company, Catalogue No. 26 of Small Tools, General Catalogue of Machinery and Tools. Providence, R. I. (From the Company.)

Canada Board of Railways Commissioners, Ninth Report, 1914. Ottawa, 1915. (From the Commission.)

Canada Department of Mines, Memoir 53, Coal Fields of Manitoba, Saskatchewan, Alberta, and Eastern British Columbia, 1914; Memoir 64, Preliminary Report on the Clay and Shale Deposits of the Province of Quebec, 1915; Memoir 57, Corundum, its Occurrence, Distribution, Exploitation, and Uses, 1915. Ottawa, 1914–15. (From the Department.)

Canadian Archives, The Canadian North-West. Ottawa, 1915. (From the Archivist.)

Cincinnati Gear Cutting Machine Company, Catalogue A of Gear Cutting Machinery. Cincinnati, no date. (From the Company.)

Connecticut State Board of Agriculture, Annual Reports, 1899, 1901, 1902, 1907 to 1913. Hartford, 1900–1914. (From the Board.)

daCunha, A., L'Annee Technique. Paris, 1905. (From the Philadelphia Book Company.)

Engineers' Society of Pennsylvania, Year Book, 1915. Harrisburg, no date. (From the Society.)

Goodrich, B. F., Company, Motor Trucks of America, vol. 3. Akron, Ohio, 1915. (From the Company.)

Indian Meteorological Department, Memoirs, vol. xix, parts v and vi. Calcutta, 1915. (From the Superintendent of Government Printing.)

Industrielle Gesellschaft von Mulhausen, Jahresbericht, 1914. Strassburg, 1915. (From the Society.)

Jeffrey Manufacturing Company, General Catalogue No. 83 of Power Transmission Machinery for the Mill, Factory, and Mine. Columbus, Ohio, no date. (From the Company.)

Kansas State Board of Agriculture, Nineteenth Biennial Report, 1914. Topeka, 1915. (From the Department.)

Lansing Company, Catalogue No. 11 of Concrete Mixers, Hoists, Skids, etc. Lansing, Mich., no date. (From the Company.)

Lewis, E. St. Elmo, Efficient Cost Keeping. Detroit, 1914. (From the Burroughs Adding Machine Company.)

Massachusetts Gas and Electric Light Commissioners, Report, 1914. Boston, 1915. (From the Commissioners.)

Massachusetts Institute of Technology, May Bulletin, Boston, 1915. (From the Institute.)

Medford Water and Sewer Department, Annual Report, 1914. Medford, Mass., 1915. (From the Department.)

National Association of Cotton Manufacturers, Transactions, 1914. Boston, 1915. (From the Association.)

National Twist Drill Tool Company, Catalogue No. 9 of Carbon and High-speed Twist Drills. Detroit, no date. (From the Company.)

New Hampshire Public Service Commission, Reports, vol. 4, 1914. Concord, no date. (From the Commission.)

New York Public Service Commission for the First District, Annual Report, 1913, vol. ii; 1914, vol. i. Albany, 1915. (From the Commission.)

New York Public Service Commission, Second District, Annual Report, 1913. Albany, 1914. (From the Commission.)

Ohio Industrial Commission, Report No. 4 of the Department of Investigation and Statistics. Columbus, 1915. (From the Commission.)

Ontario Agricultural College and Experimental Farm, Fortieth Annual Report 1914. Toronto, 1915. (From the Department of Agriculture.)

Ontario Minister of Agriculture, Report, 1914. Toronto, 1915. (From the Minister.)

Ontario Women's Institutes, Report, part i, 1914. Toronto, 1915. (From the Department of Agriculture.)

Pennsylvania at Gettysburg, Fiftieth Anniversary Report of the Pennsylvania Commission, 1913. Harrisburg, 1914. (From the State Librarian.)

Pennsylvania State Treasurer, Annual and Detailed Reports, 1914. Harrisburg, 1915. (From the State Librarian.)

Pennsylvania Topographic and Geologic Survey, Reports, Nos. 7, 8 A, 8 C, 8 E, 1911; Biennial Report, 1914; Report No. 2, 1914. Harrisburg, 1914. (From Mr. Edward V. d'Invilliers.) Report No. 7. Harrisburg, 1913. (From the State Librarian.)

Philadelphia Free Library, Nineteenth Annual Report, 1914. Philadelphia, no date. (From the Library.)

Rensselear Polytechnic Institute Publications, Engineering and Science Series, 1, 3, 4, 5, 6, 7, no date; Catalogue. Troy, 1915. (From the Institute.)

Rhode Island Public Utilities Commission, Third Annual Report, 1914. Providence, 1915. (From the Commission.)

Royal Irish Academy, Proceedings, vol. xxxi, March and April, 1915. Dublin, Ireland, 1915. (From the Academy.)

Royal Society of South Australia, Transactions and Proceedings, vol. xxxviii, 1914; Memoirs, Chiastolites from Bimbowrie, South Australia, 1911. Adelaide, 1915. (From the Society.)

Safety on the Boston Elevated Railway. Boston, no date. (From Robert D. Jenks, Esq.)

Society of Naval Architects and Marine Engineers, Transactions, 1914. New York, no date. (From the Society.)

State College of Washington, Catalogue. Pullman, 1915. (From the College.)

St. Louis University, Bulletins and Catalogue. St. Louis, Mo., 1915. (From the University.)

Stone & Webster Engineering Corporation, Building Construction. Boston, no date. (From the Corporation.)

The Transit. Iowa City, 1915. (From the College of Applied Science, University of Iowa.)

United States Geological Survey, The Pleistocene of Indiana and Michigan and the History of the Great Lakes. Washington, 1915. (From the Survey.)

United States War Department, Annual Report of the Chief of Engineers, vol. ii. Washington, 1914. (From the Department.)

University of Arizona, Annual Catalogue. Tucson, 1915. (From the University.)

University of Idaho, Catalogue. Moscow, 1915. (From the University.)

University of Louisville, Catalogue. Louisville, 1915. (From the University.)

University of Michigan, Astronomical Observatory, Publications, vol. i, pp. 73–206. Ann Arbor, 1915. (From the University.)

University of Missouri Bulletin, Science Series, vol. 1, Nos. 1–9; vol. 2, Nos. 2, 3, 4. Columbia, 1911–1914. (From the University.)

University of Nebraska, General Catalogue. Lincoln, 1915. (From the University.)

University of Texas, Catalogue. Austin, 1915. (From the University.)

University of the State of New York, Education Department, Annual Report, 1914; Proceedings of the Inauguration of John Huston Finley, 1914. (From the State Librarian.)

University of Toronto, Calendar of Department of Applied Science and Engineering. Toronto, 1915. (From the University.)

Verein Deutscher Chemiker, Mitgliederverzeichnis, 1915. Leipzig, no date. (From the Society.)

Ware Brothers Company, The American Fertilizer Hand Book, 1915. Philadelphia. (From the Company.)

Wisconsin Geological and Natural History Survey, Bulletin No. 34, Limestone Road Materials of Wisconsin. Madison, 1914. (From the Survey.)

PUBLICATIONS RECEIVED.

U. S. War Department, Annual Report, 1914: Reports of the Secretary of War, the Chief of Staff, the Adjutant General, the Inspector General, the Judge-Advocate General, the Quartermaster General, the Surgeon General, the Chief of Ordnance, the Chief Signal Officer, the Chief of Coast Artillery, the Military Academy, Military Parks, Chief of Engineers, Chief Bureau of Insular Affairs, Governor of Porto Rico. 3 volumes, illustrations, plates, maps, tables, 8vo. Washington, Government Printing Office, 1915.

Engineers' Society of Western Pensylvania, Year Book, 1915. 95 pages, 16mo. Harrisburg, Pa., Society, no date.

Pennsylvania Topographic and Geologic Survey: Report No. 8, The Mineral Production of Pennsylvania, 1911. Coal and Coke. Petroleum and Natural Gas. Clay and Clay Products, Sand, Lime, Brick. Copper, Nickel, Iron Ore, Salt and Bromine, Abrasive Materials, Graphite, Feldspar and Quartz, Talc and Soapstone, Slate, Mineral Pigments, Cement, Sand and Gravel, Mineral Waters. Quarry Products, Granite, Trap Rock, Sandstone, Bluestone, Marble, Limestone, Lime. 275 pages, 8vo. Harrisburg, Pa., State Printers, no date.

U. S. Department of Agriculture: Bulletin 220, Road Models Prepared by the Office of Public Roads. 24 pages, illustrations, 8vo. Washington, Government Printing Office, 1915.

Safety and Short Trains, address by Marcus A. Dow, General Safety Agent, New York Central Lines. 30 pages, 12mo. No place, no date.

New York Stock Exchange in the Crisis of 1914, by H. G. S. Noble, president. 89 pages, 8vo. Garden City, Country Life Press, 1915.

Analysis of Milk and Milk Products, by Henry Leffmann, M.D. Fourth edition, revised and enlarged. 115 pages, illustrations, 12mo. Philadelphia, P. Blackiston's Son & Co., 1915. Price, $1.25.

500,000-volt Transformer of the Oerlikon Works. ANON. (*La Revue Electrique,* vol. xxiii, No. 271.)—The Oerlikon Works regularly manufactures testing transformers giving voltages of 200,000 and 250,000. For use in their own laboratories they have found it necessary to construct a transformer yielding in normal operation a difference of potential of 500,000 volts. The exterior shape of the apparatus is cylindrical, 3.3 m. in diameter and 3.6 m. high. Its total weight is 34 tons. It is capable of taking a current of 1000 ampères at 500 volts. At 500,000 volts the sparking distance is 1.32 m., and at its maximum overload, giving a voltage of 700,000, the sparking distance is 1.85 m.

CURRENT TOPICS

Precision Resistance Measurements. E. H. RAYNER. (*The Physical Society of London, Proceedings,* vol. xxvii, part iv.)—The paper describes methods by which the comparison of resistance can be made to an accuracy of 1 in 10,000 or higher by using simple apparatus usually available in electrical laboratories or which can be easily constructed with little skilled assistance. The comparison of nominally equal resistances of one ohm and upwards by the usual method of shunting one side of a nearly balanced quadrilateral by a high resistance is mentioned, and variations of this when only part of one resistance is shunted are often useful. The great advantage of having resistance capable of carrying comparatively large currents is illustrated, especially for measuring changes of resistance of commercial apparatus under working conditions. The determination of errors in a volt-box for use with a potentiometer is described at some length. This is of especial importance in precision photometry.

If a sufficient continuous-current voltage is not available for testing such apparatus as high-potential dividers, it is shown that, using sufficient continuous current to secure sensitivity, the heating may be supplied by superposed alternating current.

Resistances in common use are very generally of simple numerical value, and a Kelvin bridge specially designed for the comparison of such resistances is described. It consists essentially of two rows of 25 resistances of 20 ohms each.

The Change in Thermal Conductivity of Metals on Fusion. A. W. PORTER and F. SIMEON. (*The Physical Society of London, Proceedings,* vol. xxvii, part iv.)—The change in question was determined for mercury and sodium by finding the temperature at different points of a cylinder of the metal contained in a glass tube. The ends of the cylinder were maintained at such temperatures that the metal was liquid half way down its length, the remaining part being solid. The temperatures were taken by means of thermo-electric junctions inserted in narrow tubular depressions which had been formed in the glass tube by forcing a knitting needle down into the locally-heated glass. The ratio of the thermal conductivity for solid and liquid was estimated from the slope of tangents drawn to the temperature-curve on each side of the melting-point. The values of these ratios are of the same order as the ratio of the corresponding values of the electrical conductivities. The mean value for mercury is 3.91 and for sodium 1.31. A summary list is given of other data concerned with fusion.

On the Self-Induction of Solenoids of Appreciable Winding Depths. S. BUTTERWORTH. (*The Physical Society of London, Proceedings*, vol. xxvii, part iv.)—The existing formulæ for coils of this type—viz., those of Rosa and Cohen—are shown to be inaccurate, the error amounting to one-fifth of one per cent. for the best formula when the winding depth is one-tenth the diameter of the coil. For greater winding depths the error is larger. The inaccuracy in Rosa's formula is due to the neglect of curvature in correcting for thickness, while in Cohen's formula the error is due to the approximate method of development.

New formulæ are developed by methods which are free from such approximation and which apply to any coil for which the length is greater than twice the diameter and the winding depth less than one-tenth the diameter. These formulæ are capable of giving eight-figure accuracy. Simplified formulæ are given which are suitable when only four-figure accuracy is required.

The Kodachrome Process of Color Protraiture. ANON. (The *British Journal of Photography*, vol. lxii, No. 2870.)—From time to time fragmentary reports have appeared of progress in the development of a simplified method of producing photographs in color by the now well-known color filter process in the hands of the Eastman Kodak Company. In a paper read before the Royal Photographic Society, from which the following outline is abstracted, the technical steps involved in the production of the Kodachrome transparency are given in detail:

An obvious step towards simplification in the making of prints and transparencies by the subtractive method of color photography would be the substitution of two colors for three. This has already been done in the "Kinematicolor" additive process of color cinematography where two negatives are used—a red and a green—and positives from these are projected alternately upon the screen, through red and green filters. The color rendering in this process is quite pleasing, although the correctness is much inferior to that of the three-color process, but the results obtained are sufficiently encouraging to justify the belief that a slight improvement in color rendering over the results obtained by this two-color additive process would be quite satisfactory for many purposes.

The method adopted is to make the negatives on specially-prepared panchromatic plates and through the correct filters, and then to develop them as usual. They are then chemically treated so as to remove the black silver and leave the plate looking just like a colorless sheet of gelatine, showing no sign of an image. When this plate is put into the specially-prepared dye bath the dye goes into the gelatine most easily when the silver is absent in the negative; that is, when there is least light in the original photograph or in the part represented by deep shadows, while in the parts corresponding to the highlights, when there is much silver in the negative, the dye penetrates

more slowly, so that as the dye slowly enters the film the original negative is transformed into a positive produced in a colored dye. While one color is made in this way, the companion picture is also dyed in the other color, and the two, when placed together, make the finished picture. Since only two colors are used in the process, it is obvious that all colors cannot be correctly rendered, and the colors for which the process fails are the blues, violets, magentas, and purples. Light blues appear blue-green and violets, black; magentas appear pink, and purple, dark brownish-red. On the other hand, flesh tints of all kinds and all shades of red, orange or green, grays and blacks are well rendered. All these are predominant in portraits, and the results are very satisfying for this class of work. Portraits produced by this process are in the form of transparencies, and are intended to be viewed in a special illuminator. The pictures appear at their best when placed in the special illuminator, giving much truer color rendering than when viewed by daylight.

The process cannot be used for the production of paper prints, and in most cases it is convenient to transform the original negative into the finished positive, but, if desired, duplicates can be made from contact prints of the negatives. The new process, according to its inventors, is not well adapted in its present form to outdoor subjects, especially when blue sky is included. The limitation of the process to transparencies must also be remembered. It represents, however, a highly satisfactory and practical method of obtaining beautiful color portraits having first-rate photographic quality and giving a pleasing reproduction of the colors of the original.

Operating and Test Data for Fuel-oil Engine and Generator. R. B. WHITE. (*Electrical World*, vol. 65, No. 26.)—The very great economy that is possible to effect by the employment of the fuel-oil engine in electric light and power service is strikingly shown in the report on the performance of such an engine in the plant of the Goodland (Kan.) Light and Power Company. The oil engine is 60-horse-power, of the two-stroke semi-Diesel type for crude or fuel-oil, belted from the fly-wheel to a 75-kilowatt, three-phase, 2300-volt, sixty-cycle generator, operating single-phase at 2200 volts. This unit operates in parallel with a 100-kilowatt, single-phase generator driven by a 165-horse-power single-cylinder engine. The steam is generated from coal costing $3.25 per ton delivered. The oil engine uses Kansas fuel-oil costing 2.3 cents per gallon. Before installing the oil engine the coal bill was $600 per month, whereas the present coal and oil bill does not exceed $300 per month.

The Drop-weight Method for the Determination of the Surface Tension of a Liquid. J. L. R. MORGAN. (*Journal of the American Chemical Society*, vol. xxxvii, No. 6.)—This method for the determination of the surface tension is one of the results of the extended purely experimental and statistical investigation of the relationship

between the falling drop weight and the surface tension of a liquid, reports of which have appeared from time to time from this laboratory (Chemical Laboratories of Columbia University). The drop weights considered throughout this work are those found by the aid of apparatus already described in this journal, which enables one to determine with a high degree of precision the weight of any desired number of drops of liquid falling from any sharp-edged capillary dropping tip at any fixed temperature below 90°, free from any effect of evaporation. As the result of an experimental study of some two hundred especially purified liquids and solutions, for more than one hundred of which the surface tensions (lying between 15.3 and higher than 76 dynes per centimetre) for purposes of comparison are known from some other method, of liquids which include those of every conceivable type and possessing every property which might adversely influence their behavior, it has been found that under certain definite conditions the drop weights of liquids from any one dropping tip are proportional to their surface tensions at the same temperature.

Potash from Kelp. (*Commerce Reports*, 1915, No. 143.)—The Bureau of Soils in the Department of Agriculture has issued a most timely and important monograph on the kelp beds in the Pacific, found off a large proportion of our littoral, including Alaska. The report in question bears the title "Potash from Kelp," and is prepared by Dr. Frank K. Cameron, who has the supervision of chemical, physical, and fertilizer investigations. It presents in compact form the results of an enormous amount of marine exploration and analytical work in the laboratory. Nothing has been omitted which is requisite for an adequate knowledge of the extent of the beds, the local characteristics, the percentage of potash and other valuable constituents present, the facilities for exploitation, and all other factors which may eventually come into play in any attempt to utilize on a generous scale the great possibilities of this magnificent national asset.

The Action of Certain Colloids on Ions during Electrolysis. A. MUTSCHELLER. (*Metallurgical and Chemical Engineering*, vol. xiii, No. 6.)—As early as 1885 it was recognized that the addition of certain substances to electroplating and refining baths had specific effects on the form and consistency of the deposit upon the cathode. Selections from those substances which improve the deposit were recognized to be colloids and, as a rule, of the amphoteric or reversible type. A further suggestion was found in the fact that the temperature coefficients on the rate of change of the migration velocity of the ions are such that they tend to make the velocity of the two ions equal as the temperature rises. Moreover, the form of the deposit itself, such as it is obtained from a sure electrolyte in the form of striæ or branches, was a sufficiently powerful inducement to a hypothesis that the regulation of the velocity of the ions should constitute the influencing factor in question. The experimental investigation

leads to the general results that, if to a given concentration of an electrolyte, gelatine solution is added in a small quantity, the velocity of migration of the anions is decreased and gelatine, together with the absorbed anions, migrates to the anode. On increasing this addition, a point is reached where the gelatine-anion complex ceases to migrate, while the relative migration velocity of the cations has become equal to 1. If finally this addition is further increased, it is found that the complex gathers on the other electrode, the cathode, where it deposits with the metal.

Optical Pyrometers. C. R. DARLING. (From "The Measurement of High Temperatures," *The Engineer,* vol. cxviii, No. 3078.)—In connection with the recent work on radiation pyrometers and their calibration, it is of interest to recall the views of Sir Robert Hadfield on the subject:

Measurement of temperature from a distance by means of radiation or optical pyrometers fails for two chief reasons: *First,* the temperature of the surface on which the instrument is sighted is lower than that of the interior, even in the case where no oxidation occurs and no slag is present. In practice the presence of oxide or slag in layers of varying thickness renders it impossible to establish any fixed relation between the temperature of the surface and the true internal temperature, and hence concordant results can not be obtained. *Second,* when fumes are rising from the ladle, as is frequently the case with alloys, the radiations received by the instrument are seriously diminished, and the reading, even if otherwise reliable, would be too low on this account. These difficulties have been partially overcome by Whipple's modification of Féry's radiation pyrometer described by the writer in the *Engineer* for June 13, 1913, in which the pyrometer is permanently mounted at the open end of a fire-clay tube so as to be focussed on the closed end, which is inserted in the molten metal. This method, while suitable for crucibles and small ladles, can not be applied to large masses, as the readings would be affected by the intense heat existing in the vicinity of the pyrometer. The solution of the problem, therefore, appears to lie in some suitable application of the thermo-electric or resistance methods of measuring high temperatures.

Inorganic Fodder. ANON. (*Scientific American,* vol. cxiii, No. 1.)—Germany possesses abundant supplies of potatoes, beet roots, and turnips; that is, materials holding carbohydrates. Albuminous fodder, on the other hand, is available only in limited quantities. Now an invention recently made at the Institute of Fermentation Industries, Berlin, will, it seems, allow a nourishing yeast, containing more than fifty per cent. of albumin, to be prepared from sugar and ammonium sulphate. Sugar is, to this effect, bound to an organic base; in order to supply the albumin so far wanting, the sugar is "fertilized" with ammonia, potash, and magnesia in the form of

their salts, after which some yeast is introduced and a strong air current applied. The yeast then absorbs the sugar and the "fertilizer," thus resulting in the formation of a highly albuminous yeast.

The process above outlined, of course, can be used only in such countries as are amply provided with sugar and ammonia. Germany, as is well known, raises much beet sugar, while Dr. Haber's synthetic process enables ammonia in any amounts to be prepared from nitrogen and hydrogen. The dry yeast obtained according to the above process constitutes an excellent nourishing fodder for cattle, and especially for horses. It is now prepared in existing press-yeast factories, but will shortly be manufactured in large works erected for the purpose.

The Inflammable Limits of Mixtures of Gasoline Vapor and Air. G. A. BURRELL and H. T. BOYD. (*The Journal of Industrial and Engineering Chemistry*, vol. 7, No. 5.)—Gasoline is so widely used in internal-combustion motors and for other purposes that the limits of complete inflammation in mixtures of its vapor and air are of interest to users of gasoline generally. Using a 100 Cc. Kempel explosion pipette, and igniting the mixtures from the top, there was obtained as the lower limit of complete inflammation a value lying between 1.9 and 2.0 per cent. of gasoline vapor. The upper limit under these conditions was found to be between 5.2 and 5.3 per cent. of gasoline vapor. The gasoline used had a specific gravity of 73° Baumé. Under the same conditions, except ignition of the mixtures from the bottom, there was obtained a value lying between 1.5 and 1.6 per cent. of gasoline vapor or the low limit. With the same grade of gasoline, using a 2800 Cc. vessel and with ignition from the bottom, by means of an electric flash produced by pulling two wires apart through which a current of 7 ampères at 220 volts was flowing, there was obtained a value lying between 1.4 and 1.5 per cent. gasoline vapor. The high limit under these conditions lay between 6.0 and 6.4 per cent. of gasoline vapor. Appreciably different limits were not obtained with cleaners' naphtha. When the initial temperature is increased before igniting the mixtures, the low limit is gradually decreased until, with an initial temperature of 400° C., the low limit lies between 1.02 and 1.22 per cent. of gasoline vapor.

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